



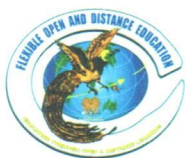
DEPARTMENT OF EDUCATION

GRADE 12 MATHEMATICS B

12.5: APPLYING GEOMETRY IN PNG ARTS



FODE DISTANCE LEARNING



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FOR THE DEPARTMENT OF EDUCATION
PAPUA NEW GUINEA



GRADE 12

MATHEMATICS B

UNIT MODULE 5

APPLYING GEOMETRY IN PAPUA NEW GUINEAN ARTS

TOPIC 1: POLYHEDRA

TOPIC 2: TESSELLATIONS

**TOPIC 3: APPLICATION IN TRADITIONAL
ARTS**



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We acknowledge the contribution of all Secondary and Upper Primary teachers who in one way or another helped to develop this Course.

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MR. DEMAS TONGOGO
Principal-FODE



Flexible Open and Distance Education
Papua New Guinea

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SECRETARY'S MESSAGE

Achieving a better future by individuals students, their families, communities or the nation as a whole, depends on the curriculum and the way it is delivered.

This course is part and parcel of the NDOE new reformed curriculum. Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the National Curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as an alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited access to secondary and higher educational institutions.

Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold:

- to facilitate and promote the integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable pathways for students and adults to complete their education, through one system, many pathways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers, who have contributed so much in developing this course.



DR. DUKE KOMBRA, PhD
Secretary for Education



UNIT INTRODUCTION

In real life, geometry has a lot of practical uses, from the most basic to the most advanced phenomena in life. Even the very basic concept of area can be a huge factor in how you do your daily business. For example, space is a huge issue when planning various construction projects.

Geometry plays a role in basic architecture design, and arts projects. For example, using the concept of shapes and tessellations can be very useful in real life.



Student Learning Outcomes

On successful completion of this module, students will be able to:

- identify prisms and pyramids
- construct nets of regular polygons and solids
- identify shapes formed by joining squares (polyominoes) and equilateral triangles (polyiamonds)
- use isometric graph paper to form tessellation patterns
- use traditional arts to identify the tessellation patterns



Time Frame

This unit should be completed within 4 weeks.

If you set an average of 2 hours per day, you should be able to complete the unit comfortably by the end of the assigned week.

Try to do all the learning activities and compare your answers with the ones provided at the end of the unit. If you do not get a particular exercise right in the first attempt, you should not get discouraged but instead, go back and attempt it again. If you still do not get it right after several attempts then you should seek help from your friend or even your tutor. Do not pass any question without solving it first.



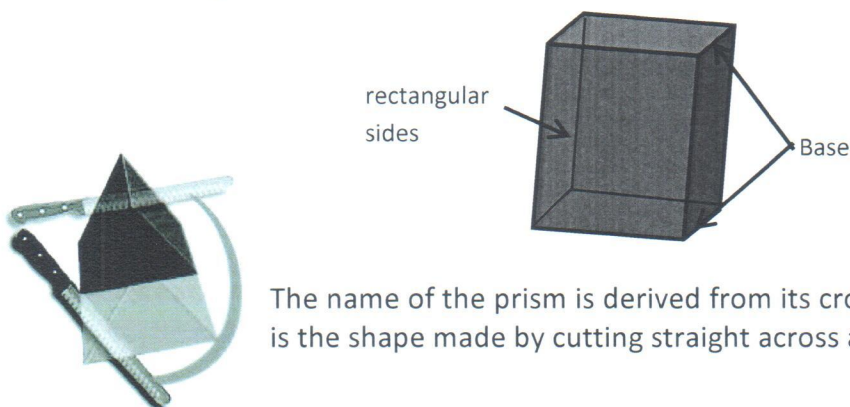
12.5.1: POLYHEDRA

Most of the time, precious gifts like rings and necklaces are given in special boxes. These vary in shapes and sizes. Some are pentagonal, hexagonal, but mostly, these boxes are cubes. The figures formed by the boxes are examples of **polyhedra**. **Polyhedra** (**polyhedron** in singular form) are solids that have polygons as their faces.

12.5.1.1: Prisms

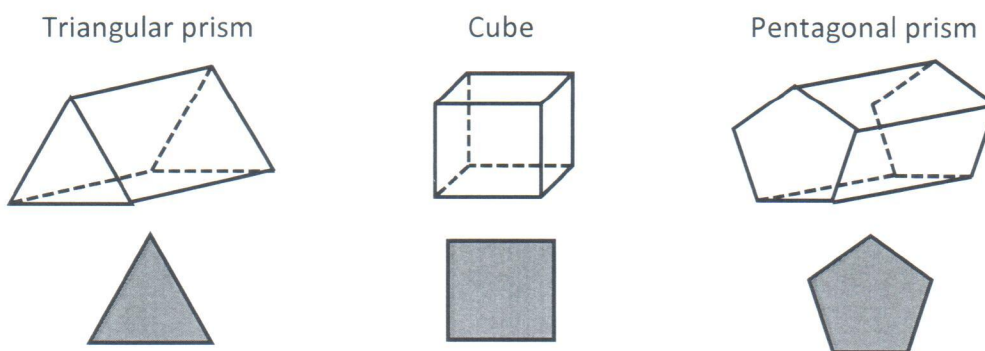
We have already studied prisms in your lower grades Mathematics.

Prisms are solids that have a constant cross-section that is a polygon with two identical bases and rectangular sides.

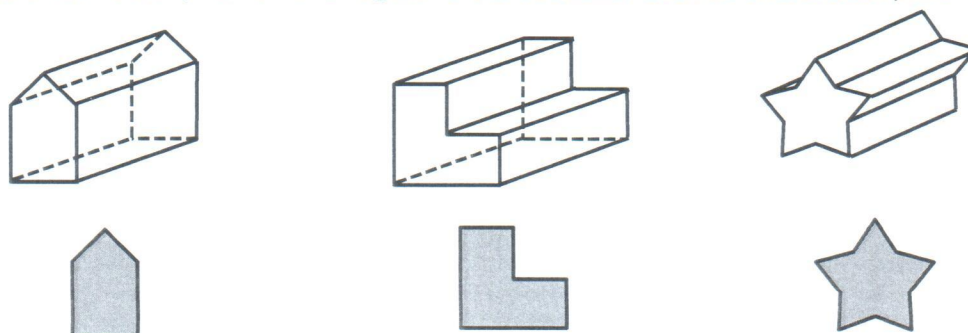


The name of the prism is derived from its cross-section. A **cross-section** is the shape made by cutting straight across an object.

Here are some examples of prisms with regular polygons as their cross sections:



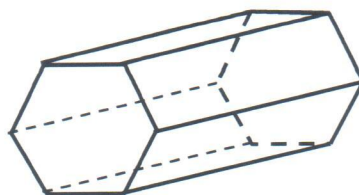
Take note that, not all prisms have regular cross sections, such as these examples:





Example 1

Draw the cross-section of the following and identify if it is a prism.



Solution:

The cross section is drawn as  which is a six-sided polygon or hexagon.

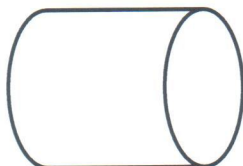
Since the cross section is a hexagon (a six-sided polygon) therefore, the given is prism.

Example 2

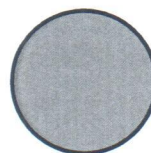
Is a cylinder considered as a prism?

Solution:

The cross section of a cylinder



is a circle



Since a circle is not a polygon, therefore, a cylinder is not a prism.

Remember that a prism is a polyhedron. This means that the sides should be polygons, whether regular or irregular. A cylinder has a circle cross-section, and a circle is NOT a polygon.

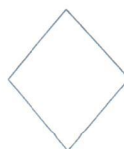
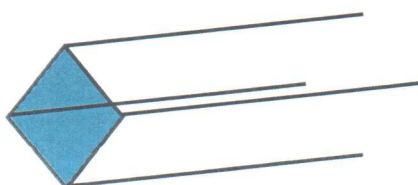
Example 3

Construct a prism with the given cross-section:



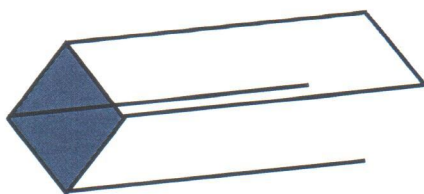
Solution:

From the edges of the polygon, draw straight lines with similar length.

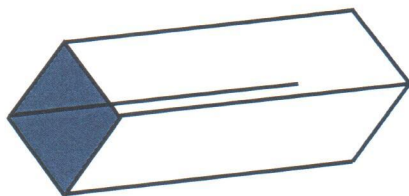




Next, connect the endpoints of the straight lines drawn to create a similar polygon with that of the given cross section.

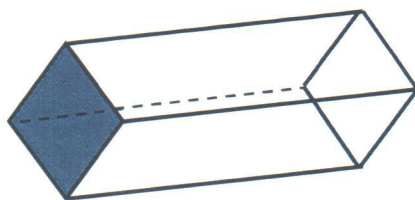


You may start connecting the two points and draw a line parallel to one side of the cross section.



Continue connecting two consecutive endpoints at a time.

Your prism will look like this.

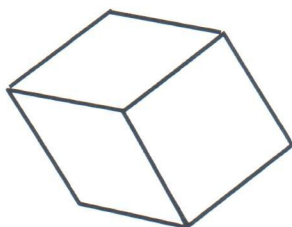


You may represent the hidden lines by using dashed lines.

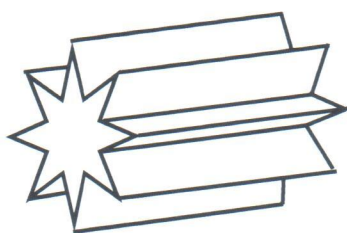
**Learning Activity 12.5.1.1****20 minutes**

1. Draw the cross-section of each of the following prisms.

a.

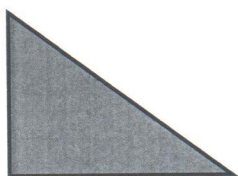


b

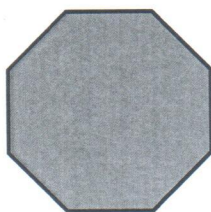


2. Construct a prism for each of the given cross-sections. Represent the hidden lines by using dashed lines.

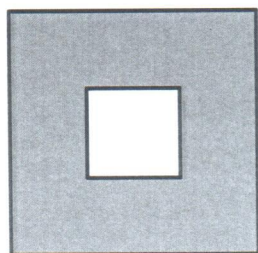
a.



b.



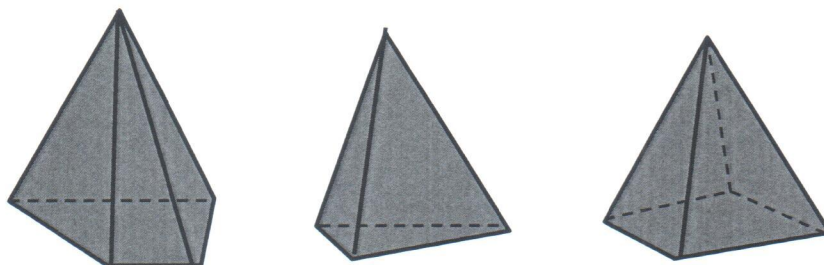
c.





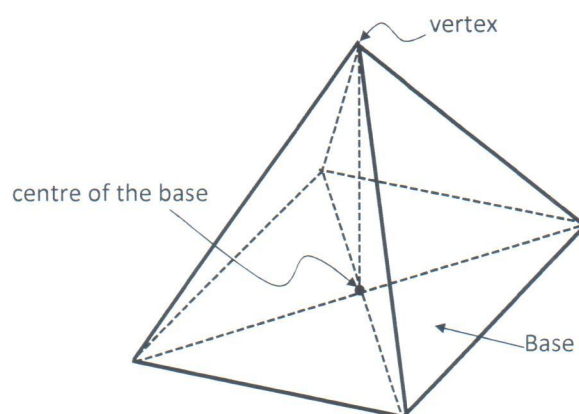
12.5.1.2: Pyramids

A **pyramid** is a solid whose base is a polygon. Each side of the pyramid is attached to a singular point called the **vertex** or **apex**. Because of this, the sides of a pyramid are triangular.

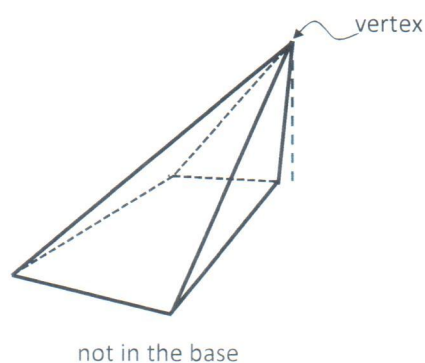
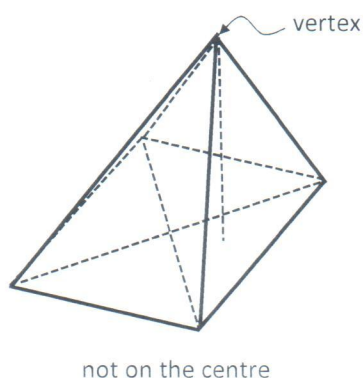


There are right and non-right pyramids.

A **right pyramid** has a vertex directly above the centre of the base such as this one:



while a **non-right pyramid** has a vertex not vertically above the centre of the base. The vertex can even be vertically above a point that is not inside the base polygon such as these:

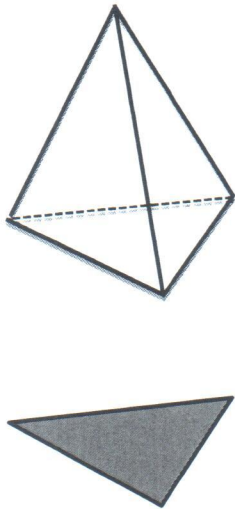




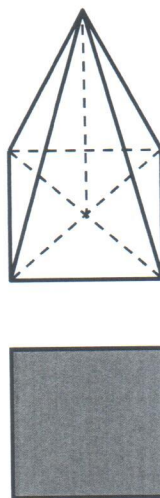
Naming pyramids is simple because they are named after the type of bases they have.

Examples are:

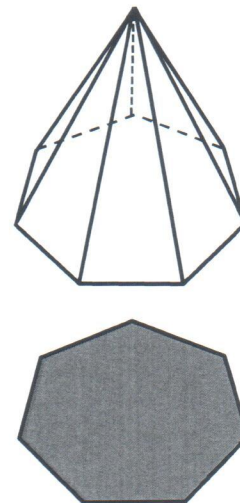
Triangular pyramid



Square pyramid

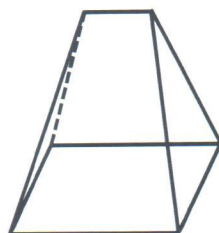


Heptagonal pyramid



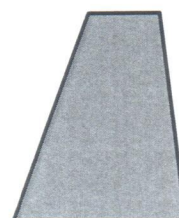
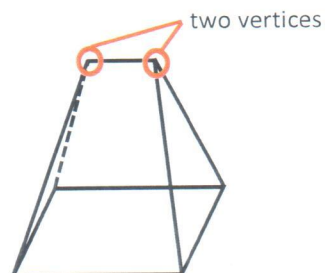
Example 1

Study the following figure and determine whether it is a pyramid.



Solution:

The figure has two vertices and two sides form a trapezoid.



Trapezoidal faces

A pyramid should only have ONE vertex with ALL sides triangular.

Therefore, the answer is NO. it is not a pyramid.

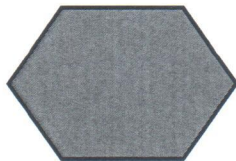


Example 3

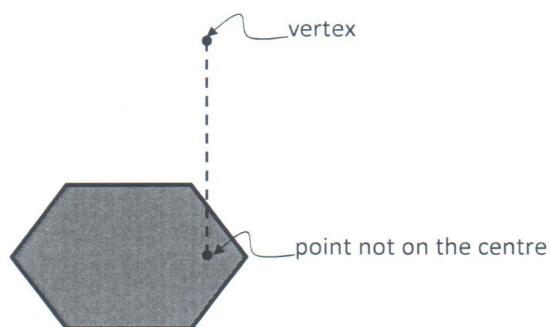
Construct a non-right hexagonal pyramid.

Solution:

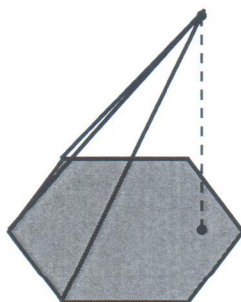
First step is to draw a hexagonal base.



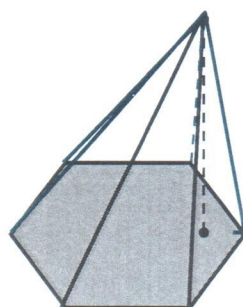
Since the pyramid to be drawn is non-right, we determine any point on the base where we will draw a straight line upwards to make the vertex.



Next is to connect the vertex to the vertices of the base using straight lines.



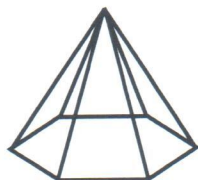
Connect all the vertices to the apex or main vertex. You may use dashed lines for hidden edges and the resulting pyramid shall be like the following:



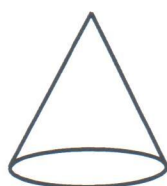
**Learning Activity 12.5.1.2****20 minutes**

1. Identify whether the following figures are pyramids by writing **P** if it is a pyramid, otherwise, **NP** on the blank provided.

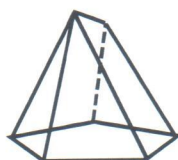
a. _____



b. _____



c. _____



2. Construct the following pyramids. Use dashed lines to represent the hidden lines.

a. a right rectangular pyramid

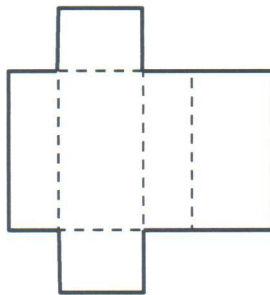
b. a non-right octagonal pyramid



12.5.1.3: Constructing Polyhedra

Remember that a polyhedron is a solid that has polygons as its faces. Based from this definition, prisms and pyramids can be classified as polyhedral. However, there are other polyhedra which can be constructed by combining regular and irregular polygons. These solids can be deconstructed to show the underlying **plane** faces of the solid. The figure formed by the connected polygons is called the **net** of the solid.

To clearly visualize this concept, deconstruct a carton box or a shoe box. Carefully tear the joining sides until the box could be laid flat. The result is the net of the deconstructed box.



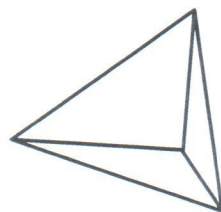
This chapter will be actual application of concepts learned through constructing figures by following simple steps. However, constructing different polyhedra requires precision in terms of the lengths and angles. To make the process easier, the net of the solid should have minimal number of edges to be joined. This means that more polygons should share the same edges with each other. Tabs for gluing could also be made to help connect the polygons and make the structure.

12.5.1.3.1 Platonic solids

Platonic solids are **polyhedra** whose faces are **regular polygons** with **congruent** sizes wherein same number of faces are connected at the vertices.

There are only five platonic solids: tetrahedron, cube, octahedron, dodecahedron and icosahedron.

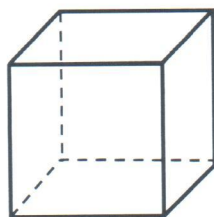
A **tetrahedron** is a polyhedron with four (4) faces that are equilateral triangles. It has six edges and four vertices (corner points) and at each vertex three triangles meet.



A tetrahedron

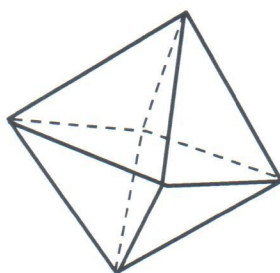


A **cube** is a polyhedron with six (6) faces that are squares. It has 12 edges and 6 vertices (corner points) and at each vertex three squares meet.



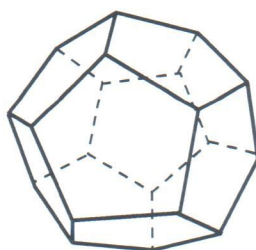
A cube

An **octahedron** is a polyhedron with eight (8) faces that are equilateral triangles. It has 12 edges and 6 vertices (corner points) and at each vertex four triangles meet.



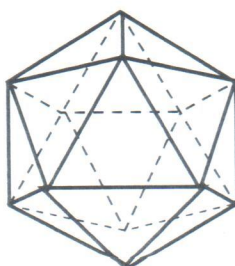
An octahedron

A **dodecahedron** is a polyhedron with twelve (12) faces that are regular pentagons. It has 30 edges and 20 vertices (corner points) and at each vertex 3 pentagons meet.



A dodecahedron

An **icosahedron** is a polyhedron with twenty (20) faces that are equilateral triangles. It has 30 edges and 12 vertices (corner points) and at each vertex 5 triangles meet.



An icosahedron



Why are there only five platonic solids?

There are a number of rules that apply to Platonic Solids:

1. The faces of each Platonic Solid are all identical regular polygons.
2. At each vertex or corner at least three faces or more must meet.

Note: Two polygons do not build a solid angle.

3. When you add up the internal angles that meet at each vertex, they must equal less than 360 degrees to get a closed solid shape.

Note: At 360 degrees, the shape would flatten out to form a plane.

A **regular triangle** has internal angles of 60° , so we can have:

- 3 triangles ($3 \times 60^\circ = 180^\circ$) tetrahedron
- 4 triangles ($4 \times 60^\circ = 240^\circ$) octahedron or
- 5 triangles ($5 \times 60^\circ = 300^\circ$) icosahedron

all of which are less than 360 degrees.

A **square** has internal angles of 90° , so there is only:

- 3 squares ($3 \times 90^\circ = 270^\circ$) hexahedron

A **regular pentagon** has internal angles of 108° , so there is only:

- 3 pentagons ($3 \times 108^\circ = 324^\circ$) dodecahedron

4. Each shape will fit perfectly inside of a sphere, all the points touching the edges of the sphere with no overlaps.

It is impossible to have more than five (5), because any other possibility would violate simple rules about the number of edges, corners and faces you can have together and anything else adds up to more than 360 degrees at a vertex or corner.

Other Interesting Facts about Platonic Solids

Hexahedron & Octahedron

- They both have the same number of edges, being 12
- The number of faces and vertices are interchanged.

Hexahedron has 6 faces and 8 vertices

Octahedron has 8 faces and 6 vertices



Dodecahedron & Icosahedron

- They both have the same number of edges, being 30
- The number of faces and vertices are interchanged
Dodecahedron has 12 faces and 20 vertices
Icosahedron has 20 faces and 12 vertices

When we can count the number of faces, vertices and edges of a polyhedron, we discover an interesting thing:

The number of faces (F) plus the number of vertices (V) minus the number of edges (E) equals 2

This can be written as an equation: $F + V - E = 2$

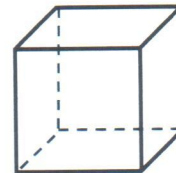
This is known as the **Polyhedral Formula** or the commonly known **Euler's Formula**.

Let us try some examples.

Example 1

The cube has: 6 faces, 8 vertices and 12 edges.

Solution: $F = 6$
 $V = 8$
 $E = 12$



Substitute these values to the formula, we have: $F + V - E = 2$

$$6 + 8 - 12 = 2$$

$$14 - 12 = 2$$

Example 2

A polyhedron has 12 faces and 20 edges, how many vertices does it have?

Solution: $F = 12$
 $E = 20$

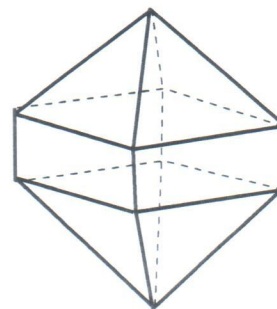
Substitute these values to the formula, we have:

$$F + V - E = 2$$

$$12 + V - 20 = 2$$

$$V - 8 = 2$$

$$V = 10$$



The figure at the right is one of the possible solid. This is an example of **non-platonic solid**.



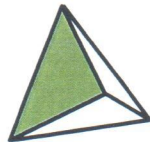
Non-platonic solid is a polyhedron form by a combination of several regular polygons or polygons that are not regular.

Constructing Polyhedron

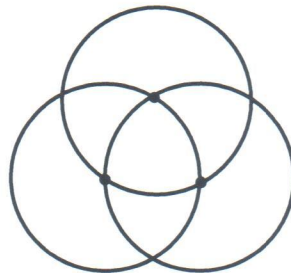
Constructing polyhedron requires a careful attention in measuring lengths and angles and it is advisable to draw a net that minimizes the number of edges that need to be connected or joined. The addition of tabs to glue the structure together also makes it easier to construct.

Example 1

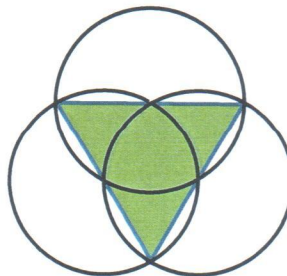
Constructing a tetrahedron



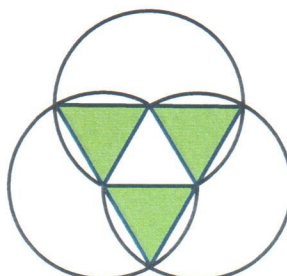
- Step 1** Using a compass draw three intersecting circles of the same radius so that the circumference of each circle goes through the centres of the other two circles.



- Step 2** Join the points of intersection on the circumference to form a large equilateral triangle.

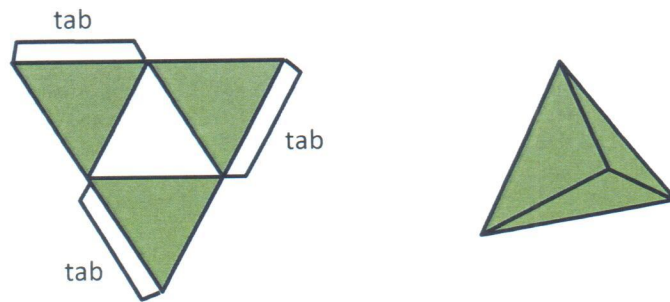


- Step 3** Join the points of intersection of the centres to form the smallest equilateral triangle.





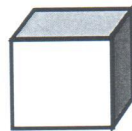
Step 4 Add some tabs for gluing your tetrahedron together to form the tetrahedron.



A tetrahedron and its net

Constructing a Cube

A **cube** is a polyhedral with six faces that are squares; eight vertices and three faces that meet at each vertex. Cubes are very common polyhedra seen in our environment. Some examples of the everyday things are jewelry boxes, sugar cubes, toys and even pots! Can you think of other objects which take the form of a cube?

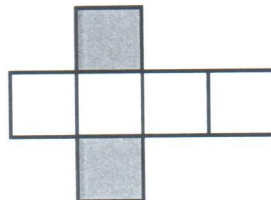


Constructing a cube is the simplest among all the polyhedra. Here is how:

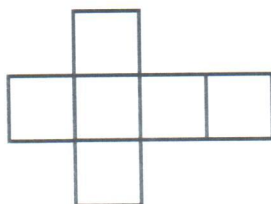
1. Draw four squares of the same sizes adjacently.



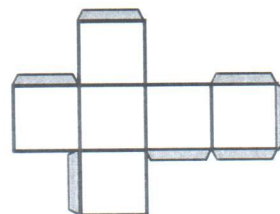
2. Draw two more squares which are not collinear to the previous four and are not adjacent to each other.



3. Add tabs outside of the squares for gluing. Cut the constructed net of the cube. Crease along the lines and glue the tabs inside of the figure.



Net of the cube

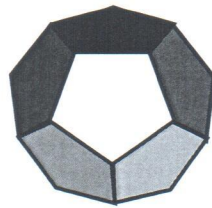


Net of the cube with gluing tabs



Constructing a Dodecahedron

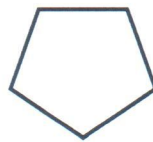
A **dodecahedron** has twelve faces, all of which are regular pentagons. This figure has twenty vertices wherein three faces meet at each vertex. This figure needs accuracy and focus in order to be created properly.



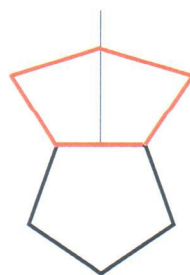
These are the steps in making a dodecahedron:

The trickiest part is constructing a regular pentagon. To be able to do this:

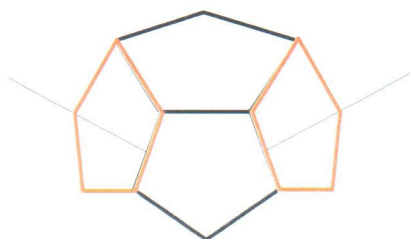
1. The basic face of this shape is a pentagon. Draw one with a horizontal top as shown below.



2. Draw another pentagon right on top of the first. Draw it from the perspective of look directly down at the first shape and seeing the second rise up from it like a pentagonal wall.

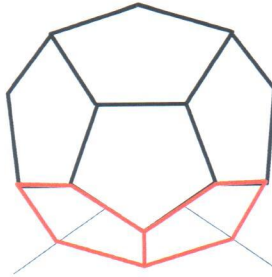


3. Draw two guidelines from the centre of the main pentagon's sides. Use these to draw the next two pentagons as shown.

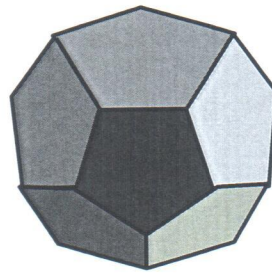




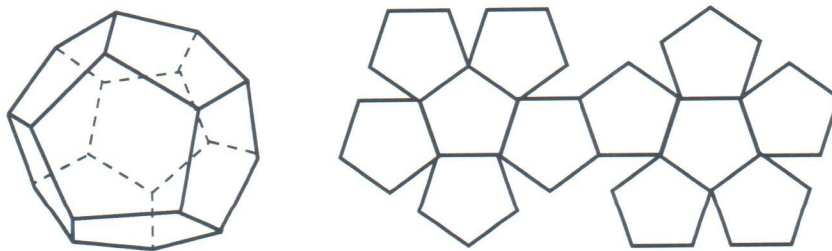
4. For the bottom faces do the same as in the previous step. This time however, the guidelines should be pointing out and tilting down.



5. To give your dodecahedron a three dimensional look, use different shades of the same colour. Colour the faces based on where the light source comes, ranging from bright to dark shades.

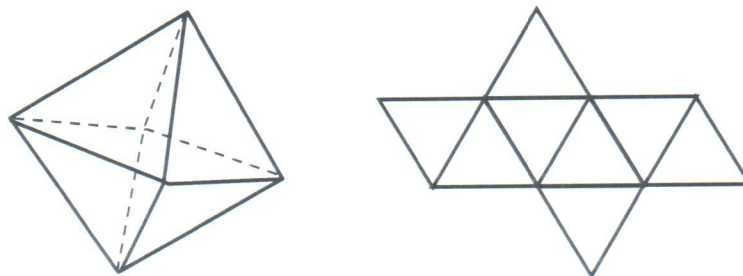


Another way of constructing polyhedrons is by drawing their nets.



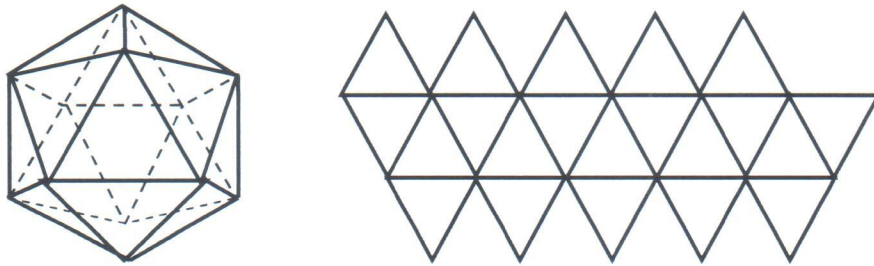
A dodecahedron and its net without tabs

Constructing Octahedron



An octahedron and its net without tabs

Use the template to draw the net.



An icosahedron and its net without tabs

Now do the learning activity.



Learning Activity 12.5.1.2



20 minutes

A. Multiple-choice

Circle the correct answers from the alternatives A, B, C, or D.

1. An octahedron has 8 faces and 12 edges. How many vertices does it have?
A. 18
B. 10
C. 6
D. 2
2. An icosahedron has 30 edges and 12 vertices. How many faces does it have?
A. 14
B. 16
C. 18
D. 20
3. A dodecahedron has 12 faces and 20 vertices. How many edges does it have?
A. 30
B. 32
C. 34
D. 36



12.5.1.4 Symmetry

Much of what we consider beautiful has symmetry and balance. Nature provides many beautiful examples of symmetry, such as the wings of a butterfly or petals of a flower. Symmetric objects have parts that are congruent.



Symmetry is when one shape becomes exactly like another if you flip, slide or turn it.

In Geometry, there are two types of two-dimensional symmetry.

1. Line or Bilateral Symmetry
2. Point or Rotational Symmetry

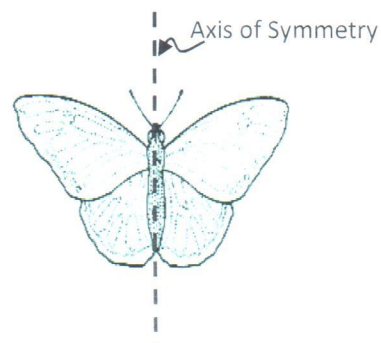
Line or Bilateral Symmetry

Line or Reflectional Symmetry exists when it is possible to draw a line through the figure so that one side is a mirror image of the other.

For example, the following objects shown are symmetrical about the dotted line. The dotted line is therefore called a **line** (or **axis**) **of symmetry**.

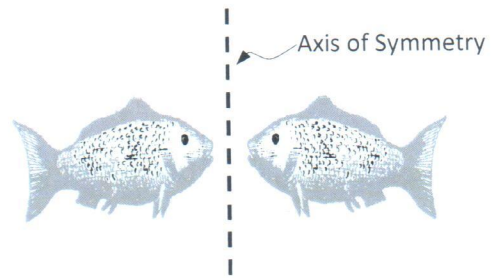
Example 1

The butterfly has line symmetry or reflectional symmetry because one side of the line is a mirror image or reflection of the other side.



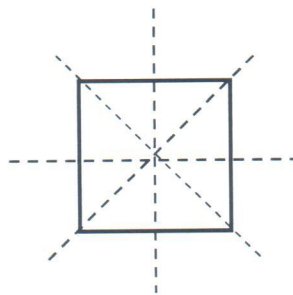
**Example 2**

The two halves of the picture are symmetrical about the dotted line. The dotted line is the axis of symmetry.



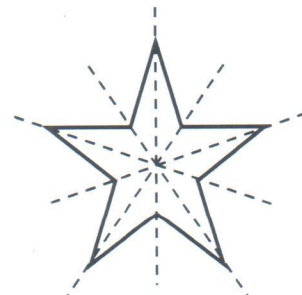
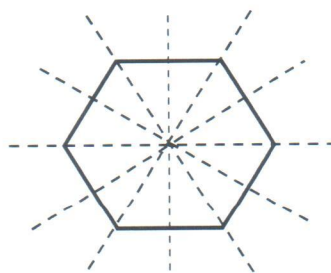
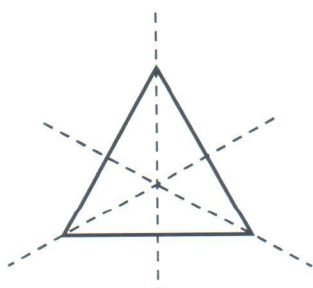
Some shapes have several lines or axes of symmetry.

For example, if you cut a perfect square in half from top to bottom, or side to side, or diagonal to diagonal, both sides are symmetrical. The square has four axes or lines of symmetry.



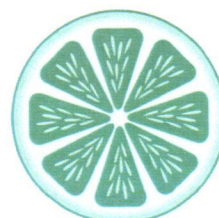
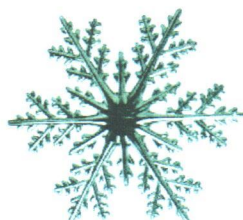
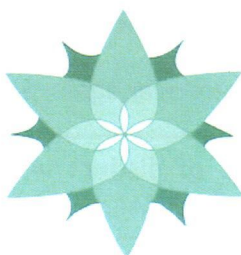
Axes of symmetry can be horizontal, vertical or diagonal.

The shapes shown below have more than one axis of symmetry.

**Point or Rotational Symmetry**

Point Symmetry or **Rotational Symmetry** is when a shape remains the same when rotated through any angle less than 360° .

Below are objects with rotational symmetry.

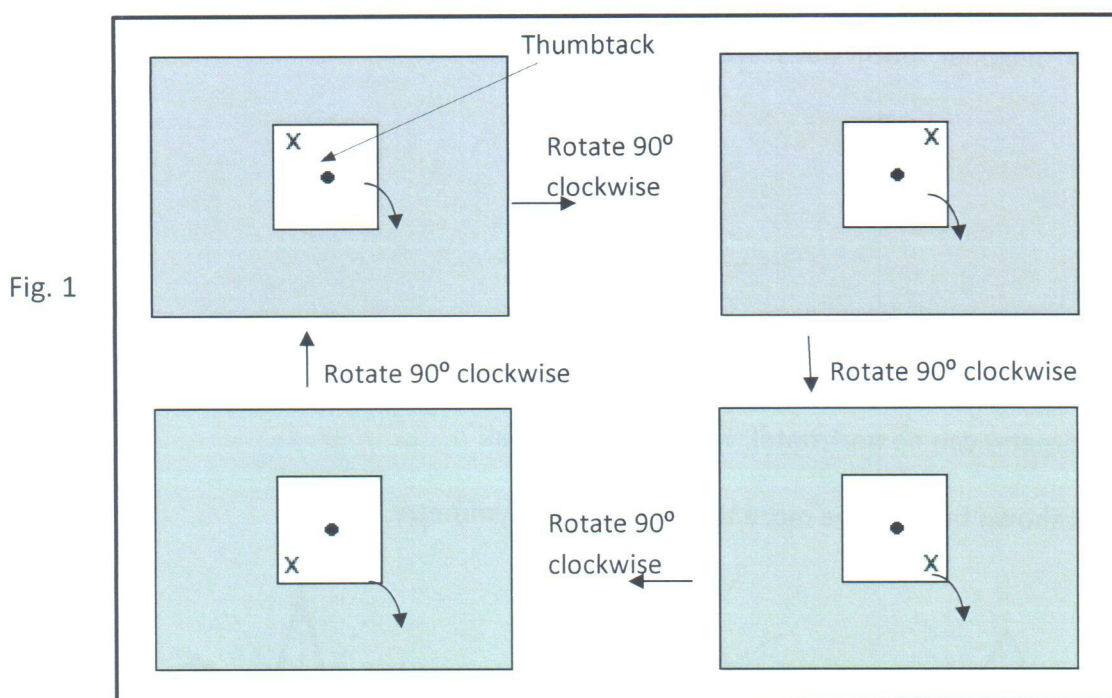


To better understand rotational symmetry let us do the following activity.

You will need the following:

Sheets of cardboard, thumbtack, pair of scissors

Cut a square shape from a sheet of cardboard. Attach the square to another bigger piece of cardboard by placing a thumbtack at the centre of the square as shown in Fig. 1. Mark a cross (x) near the corner of the square as shown below.



Trace around the shape of the square on the larger cardboard. Then rotate it 90° clockwise (or anti-clockwise) and trace around it again. The two tracings will coincide. Note the position of the cross (x) after the square is rotated 90° clockwise.

We say that the square has **Point Symmetry** or **Rotational Symmetry**. The point about which the shape rotates (thumbtack) is called the **centre of rotational symmetry**.

How many times do we have to repeat the process to get back to the original position of the square in Fig.1?

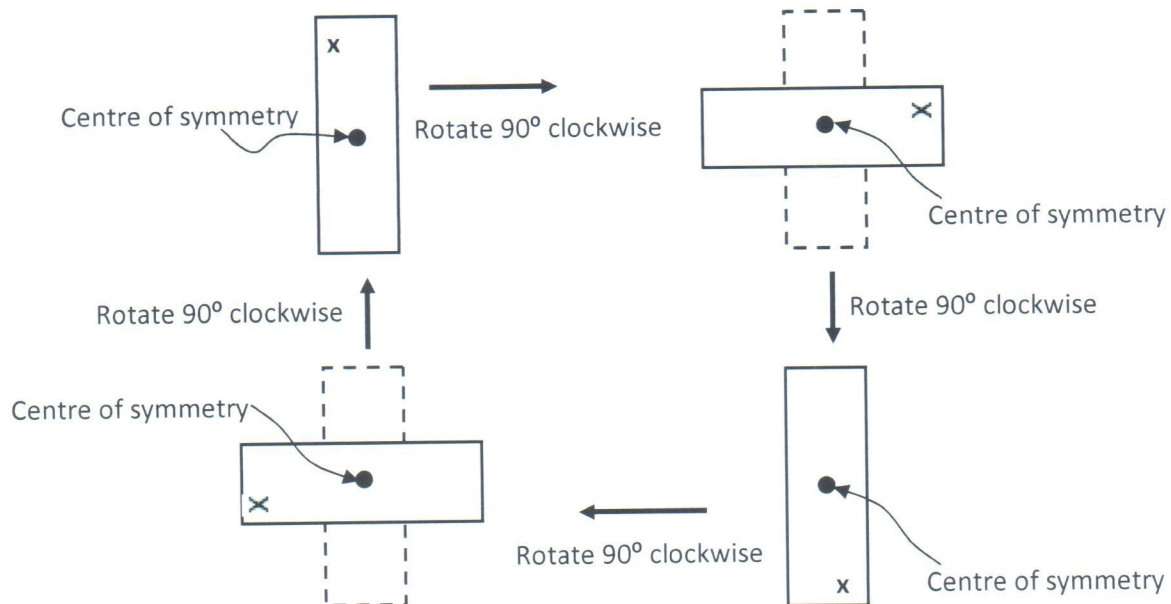
The answer is **4**. We say that the **order of rotational symmetry** is 4.

Notice that $4 = \frac{360^\circ}{90^\circ}$ or $4 \times 90^\circ = 360^\circ$. Therefore, if we repeat the process more than 4 times, the rotation goes beyond 360° , we get the same positions over again. So,

The Order of Rotational Symmetry is the number of distinct ways in which a shape or figure can map onto itself by rotation.

If you can rotate (or turn) a figure around a centre point by fewer than 360° and the figure appears unchanged, then the figure has **rotational symmetry**. The point around which you rotate is called the **centre of rotation**, and the smallest angle you need to turn is called the **angle of rotation**. The number of times the object or shape can be rotated so that it matches the original object is called the **order of rotational symmetry**.

Let us carry out the same activity illustrated by rotating a rectangular shape.



Notice that to get the tracing to coincide, we need to rotate the rectangle 180° clockwise and doing this two times, we get the rectangle back to its original position.

Therefore, the order of rotational symmetry is 2.

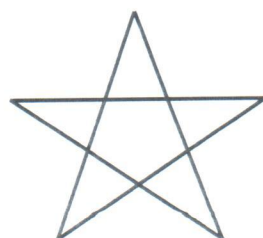
Shapes with point symmetry (half-turn symmetry) have a special property. Every point on the figure has a matching point on the figure. These points are the same distance from the centre of symmetry, and if they are joined by a line, the line passes through the centre of symmetry.

Here are some more examples.

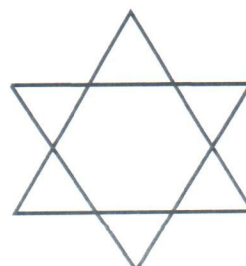
For the figures below, write down the:

1. number of lines of symmetry
2. the order of rotational symmetry

a)



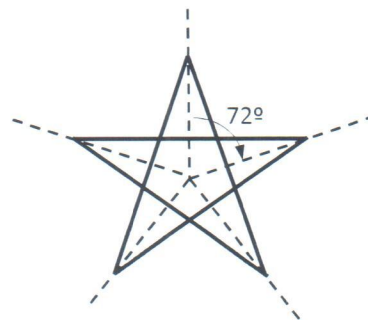
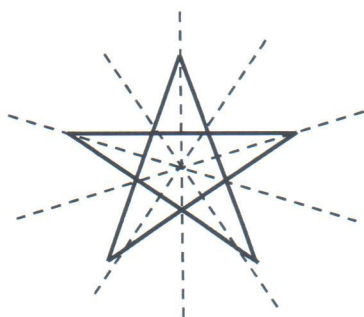
b)





Solution:

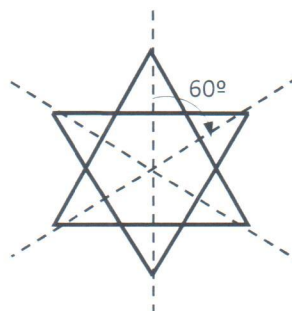
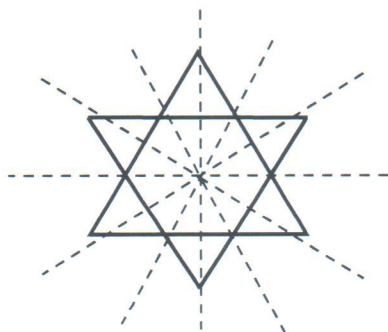
- a) 1. Number of lines of symmetry = 5
2. Order of rotational symmetry = 5



Each rotation of 72° (clockwise or counter-clockwise) will give us a distinct way in which the figure can map or fit onto itself.

$$\text{Number of distinct ways} = \frac{360^\circ}{72^\circ} = 5$$

- b) 1. Number of lines of symmetry = 6
2. Order of rotational symmetry = 6



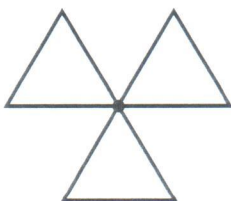
Each rotation of 60° gives us a distinct way in which the figure can map or fit onto itself.

$$\text{Number of distinct ways} = \frac{360^\circ}{60^\circ} = 6$$

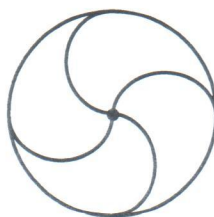
Some shapes that do not possess line symmetry still appear to have some symmetrical properties.

Look at these figures. Do these shapes have point symmetry?

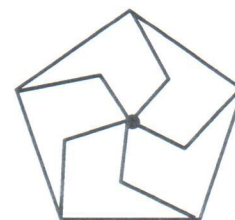
a)



b)



c.

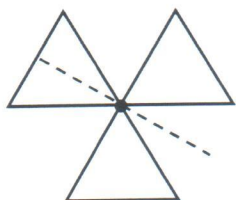




Solution:

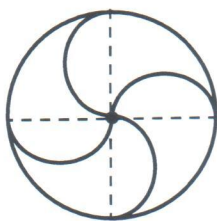
Draw a line from any point through the centre. Is there a matching point on the other side of the centre the same distance away? If the answer is yes for every point on the figure, it has point symmetry.

For figure **a**, the answer is **no**.



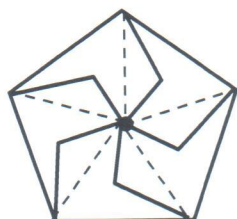
No matching point on the other side of the centre the same distance away.

For figure **b**, the answer is **yes**.



It is because in one complete rotation, the tracing fits four times. It is said to have rotational symmetry of order 4.

For figure **c**, the answer is **yes**.

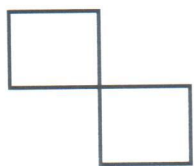


It is because in one complete rotation, the tracing fits five times. It is said to have rotational symmetry of order 5.

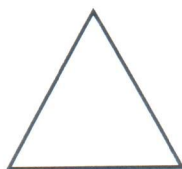
Below are some more examples of shapes and their order of rotational symmetry.



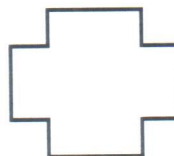
No rotational symmetry
(order 0)



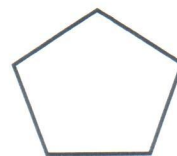
order 2



order 3



order 4

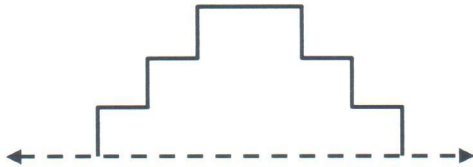


order 5

**Learning Activity 12.5.1.3****20 minutes**

1. Complete each figure. The dashed line is the line of symmetry.

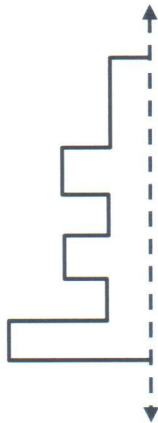
a)



b.



c.

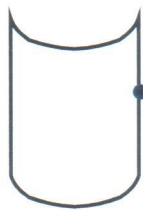


d.



2. Complete each figure. The point is the centre of rotation.

a.



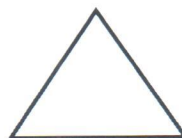
b.





3. For the equilateral triangle shown, write:

- the number of lines of symmetry
- the order of rotational symmetry.

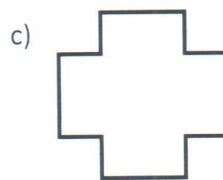


4. State the order of rotational symmetry of each shape below.

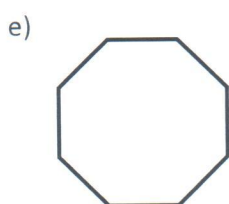


Answers: _____



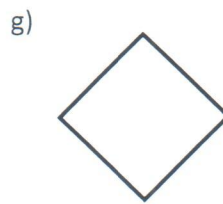


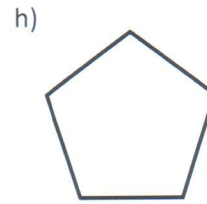


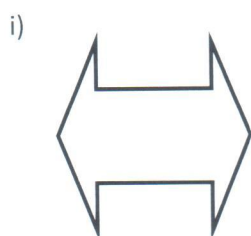


Answers: _____

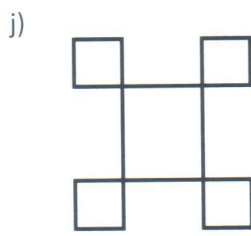


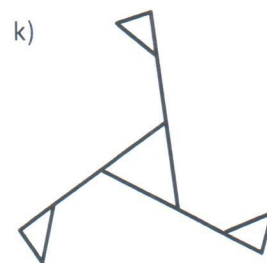


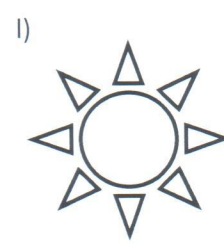


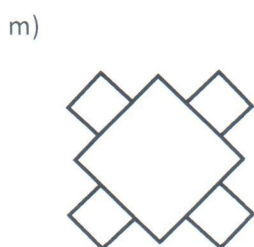


Answers: _____

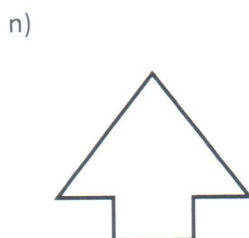


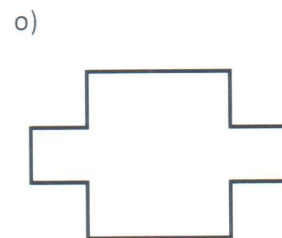


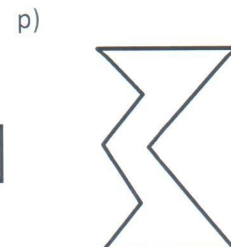




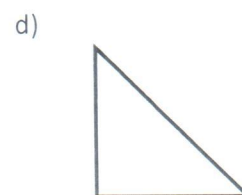
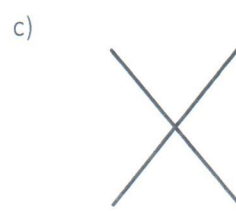
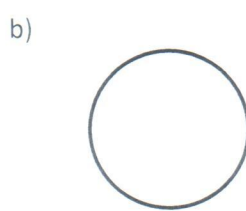
Answers: _____







5. Which of the following shapes have rotational symmetry?



Answers: _____



12.5.1.4 Polyominoes

Polyominoes are a major topic in recreational mathematics and in the field of geometric combinatorics. Polyominoes have connections with various themes in geometry such as symmetry, tiling, perimeter and area.

Polyominoes are shapes that are made up of **squares** joined edge to edge.

You can make polyominoes by using interlocking cubes. Be sure that when the figure is laid flat all the cubes are in the same plane.

Polyominoes are named according to how many cells they have.

1. Monomino – made up of a single square.



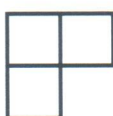
2. Domino – is made of two squares, joined edge to edge.



3. Triomino- is made of three squares.

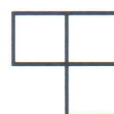


This a straight triomino

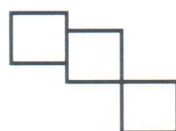


This a bent triomino

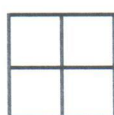
There are only two different triominoes. The shapes below are the same one as the ones above but in different positions.



However, this one below is not a triomino, since its squares are not joined edge to edge.



4. Tetromino is made of four squares. There five different tetrominoes.



Square



L



N



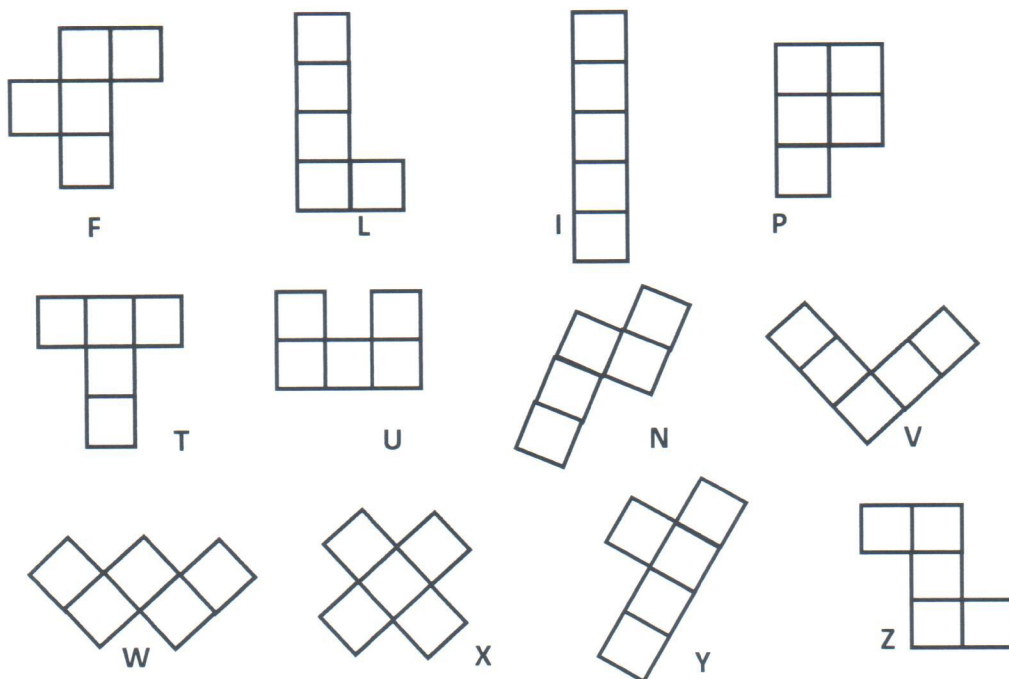
I



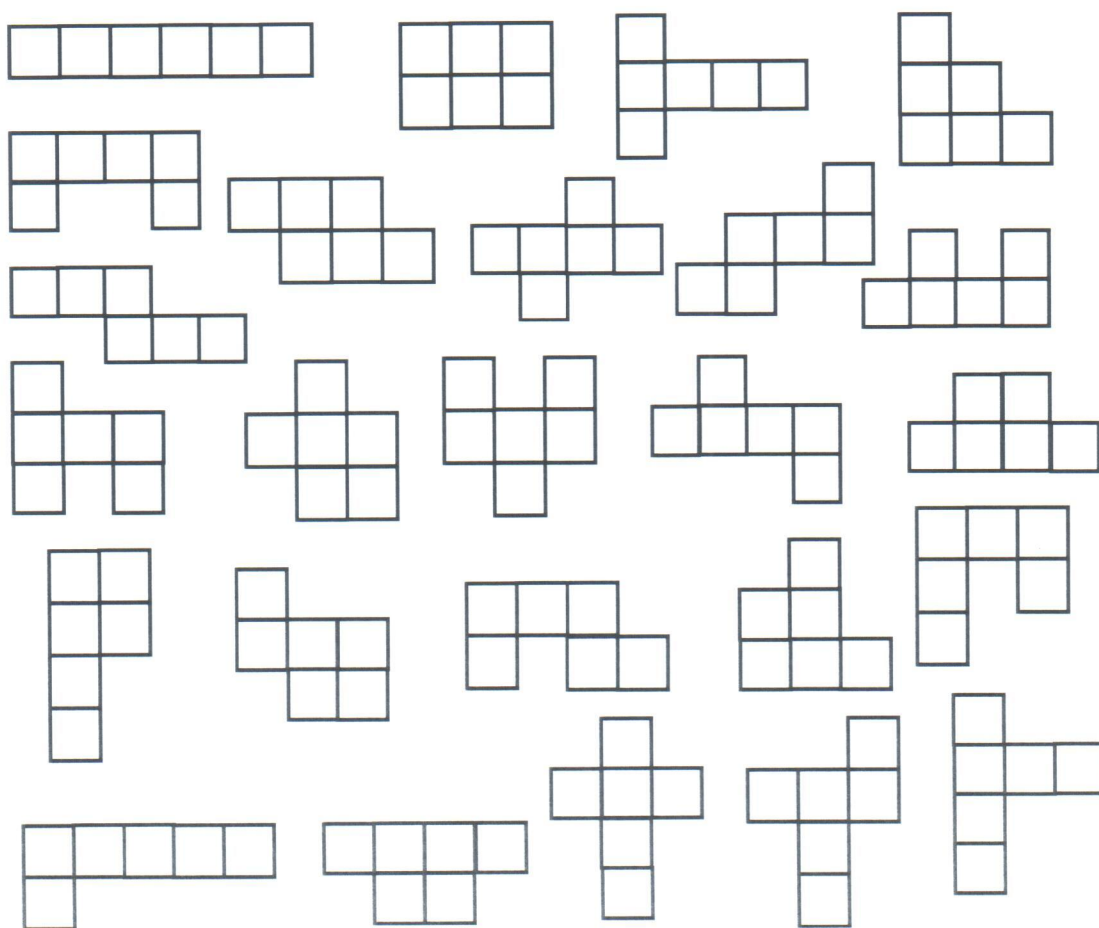
T



5. Pentomino is made of five squares. There are 12 different pentominoes.



6. Hexomino – is made of six squares. There are thirty-five (35) different hexominoes.





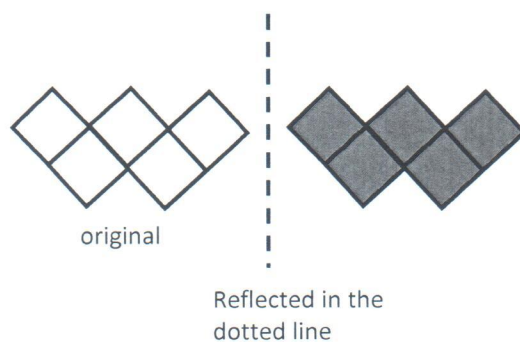
Only 25 of the 35 hexominoes are given above. Try finding and drawing the remaining 10.

Two polyominoes are the same if they can be translated (slide sideways, or up and down), reflected (as in a mirror) or rotated (turned around) so that they look the same.

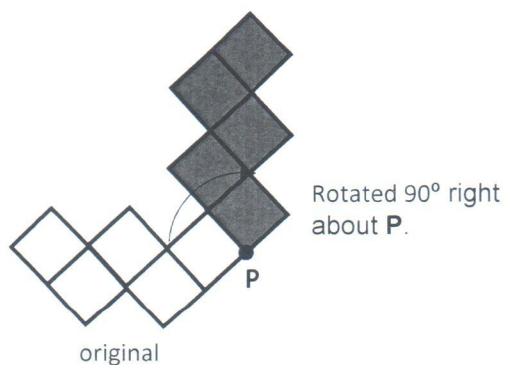
Example 1



Example 2



Example 3

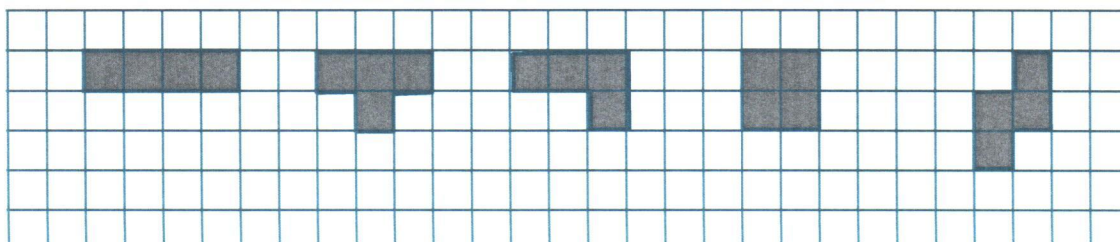


Learning Activity 12.5.1.4



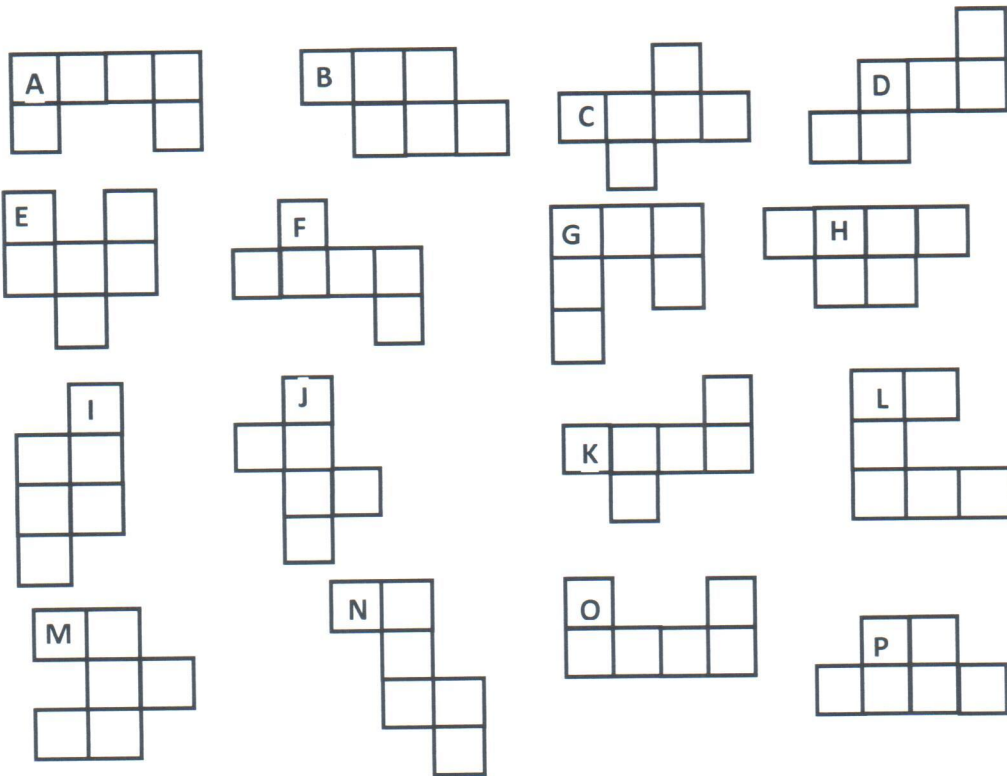
20 minutes

1. Using a Grid paper, draw the five different tetrominoes. Shade in the square in each tetrominoes.

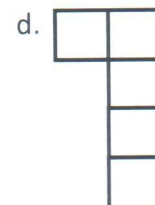
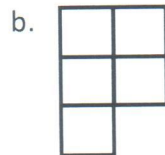
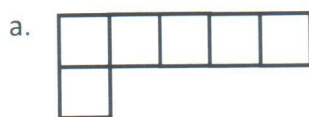




2. Below are 8 of different distinct hexominoes. Match the pairs that are the same



3. In each of the following give its perimeter and area (the sides of the squares are all 1 cm).



**12.5.1.5 Polyiamonds**

Polyiamonds are shapes that are made of **equilateral triangles** joined edge to edge.

The triangles in forming the polyiamonds are arranged in such a way that they are connected side by side and that all are in the same plane. Polyiamonds are not three-dimensional arrangement of triangles.

Polyiamonds are named according to the number of triangles they have.

Below is a table showing the name of the polyiamond and the number of triangles and the number of different arrangements of the triangles.

Name of Polyiamond	Number of Triangles	Number of Different Arrangements
Moniamond	1	1
Diamond	2	1
Triamond	3	1
Tetramond	4	3
Pentiamond	5	4
Hexiamond	6	12
Heptiamond	7	24
octiamond	8	66

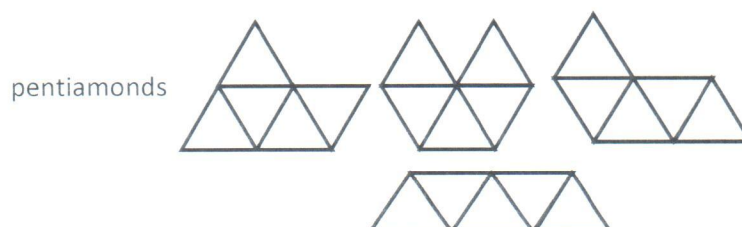
There is only one each of moniamond, diamond and triamond. This means that you can only form one figure with one, two or three triangles.



There are four figures of four triangles, the tetramonds.



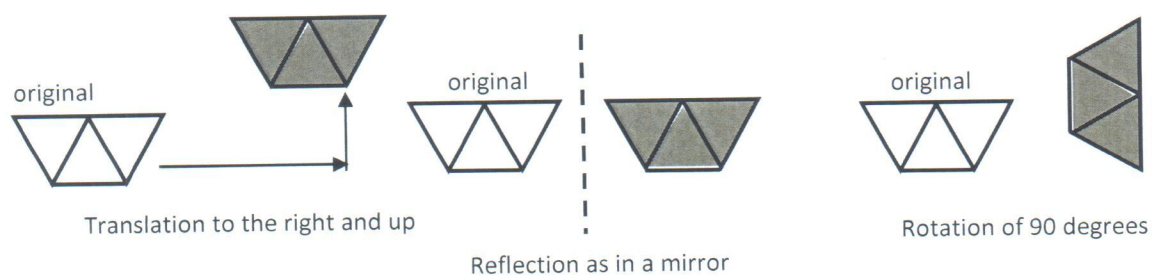
There are four figures of five triangles, the pentiamonds.



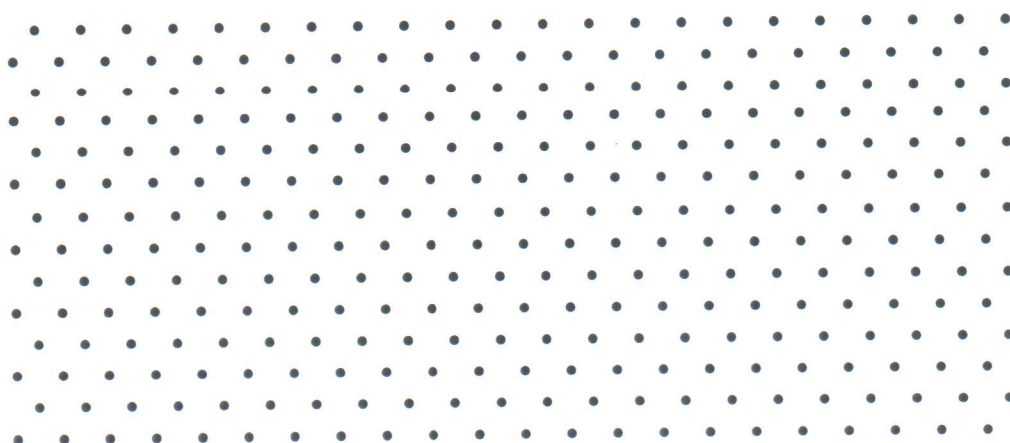
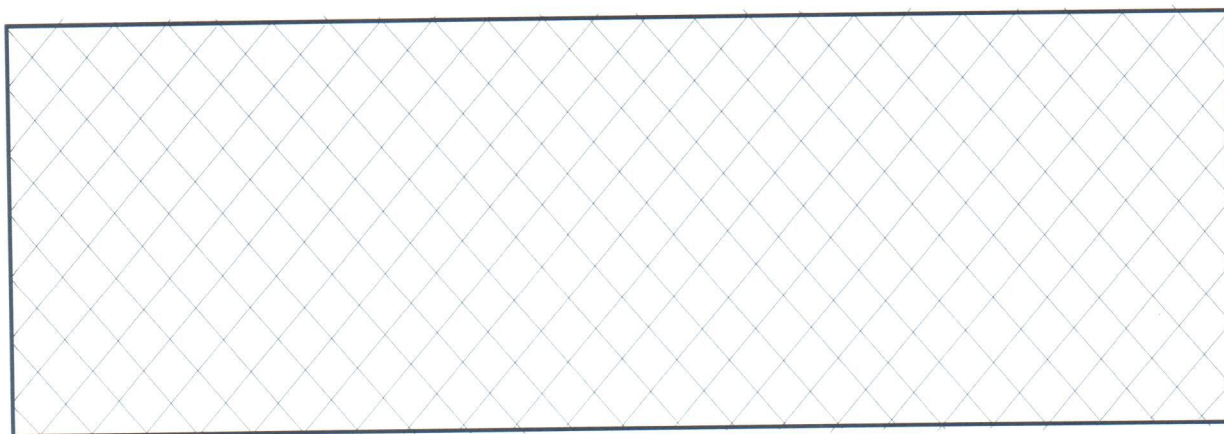


Two polyiamonds are the same if they can be translated (slide sideways, or up and down), selected (as in a mirror) or rotated (turned around) so that they look the same.

For example, all the shaded polyiamonds are the same polyiamonds as the original.



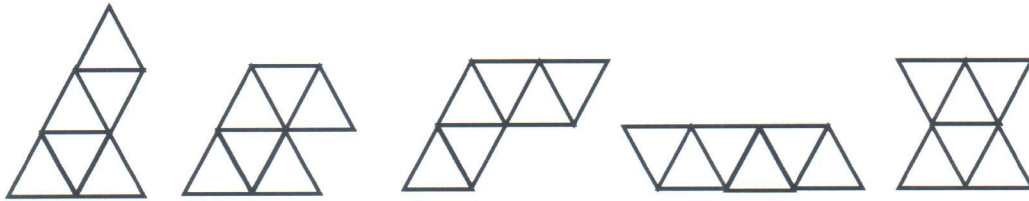
We can usually draw and shade polyiamonds using the isometric grid or dot paper.



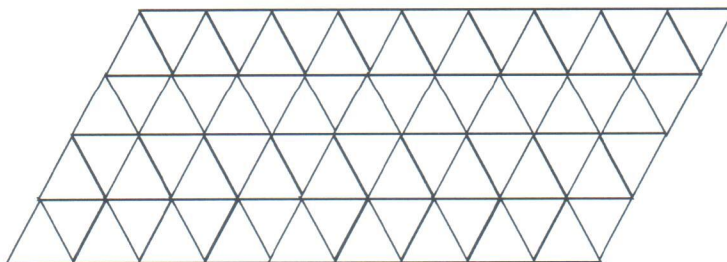
**Learning Activity 12.5.1.5**

20 minutes

1. There are 12 hexiamonds. Five are given below. There are seven more.



2. Using the parallelogram below fit all the 12 hexiamonds.



**SUMMATIVE TASK 12.5.1**

50 minutes

A. Fill in the blanks:

1. A cube has _____ vertices, _____ edges and _____ faces.
2. The point at which three faces of a figure meet is known as its _____.
3. A cuboid is also known as a rectangular _____.
4. A triangular pyramid is called a _____.
5. A polyhedron is said to be _____ if its faces are made up of regular polygons.
6. A _____ is a polyhedron whose base is a polygon and whose lateral faces are triangles with a common vertex.
7. A _____ is a polyhedron whose base and top are congruent polygon and whose other faces are parallelogram.
8. In polyhedron relationship between no. of faces (F), vertices (V) and edges (E) is denoted by $F + V - E = 2$. It is also called the _____.
9. _____ are shapes that are made up of squares joined edge to edge.
10. _____ are shapes that are made of equilateral triangles joined edge to edge.

B. Use the Euler's Formula to answer the following:

1. Write down the number of faces of each of the following figures:
 - (i) Cuboid _____
 - (ii) Cube _____
 - (iii) Triangular prism _____
 - (iv) Square pyramid _____
 - (v) Tetrahedron _____
2. Write down the number of edges of each of the following figures:
 - (i) Tetrahedron _____
 - (ii) Rectangular pyramid _____
 - (iii) Cube _____
 - (iv) Triangular prism _____



3. Write down the number of vertices of each of the following figures:

(i) Cuboid

(ii) Square pyramid

(iii) Tetrahedron

(iv) Triangular prism

4. What polygon do you see on the faces of the

(i) cube?

(ii) Tetrahedron?

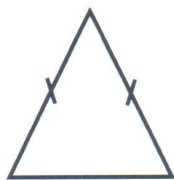
(iii) octahedron?

(iv) Icosahedron?

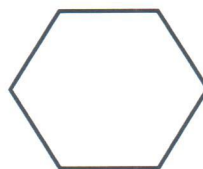
(v) dodecahedron?

C. State the order of rotational symmetry in each of the following figures.

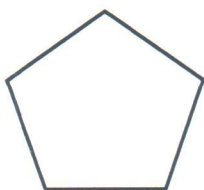
1.



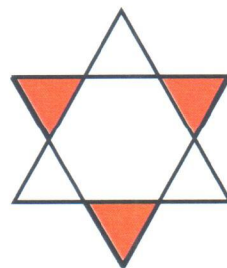
2.



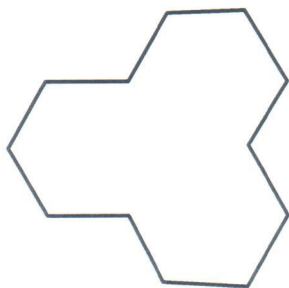
3.



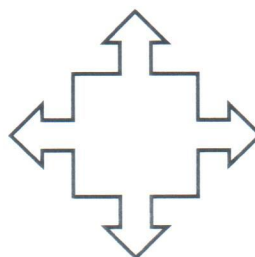
4.



5.



6.





D. Multiple-choice.

Circle the letter of the correct answer in each of the following statements.

1. Which of the following is not a platonic solid?

A. tetrahedron	B. octahedron
C. icosahedron	D. Rhombohedron

2. Which of the following shapes can NOT be fit together to make a regular polyhedron?

A. Octagon	B. pentagon
C. rectangle	D. square

3. What is the other name for a cube?

A. tetrahedron	B. octahedron
C. icosahedron	D. hexahedron

4. Another name for a corner or a point is

A. face	B. edge
C. vertex	D. polygon

5. How many platonic solids are there?

A. 4	B. 5
C. 6	D. 7



12.5.2 TESSELLATIONS

Have you ever seen a mosaic art in walls or floors of buildings? A mosaic is a picture or design made with small pieces of coloured material such as glass or tile stuck onto a surface. These pieces of materials fit together to cover the canvass without spaces and overlaps. If all the bits used are identical to form a design, the resulting pattern is a tessellation.

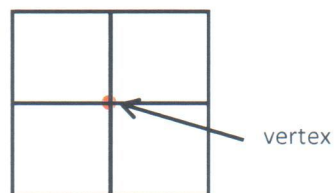
Tessellation is a pattern of polygons, each of which fit together around a common point called the vertex. A less technical term is **tiling**. When each of the polygons around the vertex are exactly the same, a tessellation is regular. However, there are also other tessellations considered are semi-regular and irregular tessellations. These are possible by combining two regular polygons (for semi-regular), and irregular polygons and shapes (for irregular) which could cover a plane endlessly without spaces and overlaps.

12.5.2.1: Regular tessellations

A regular tessellation means a tessellation made up of congruent regular polygons.

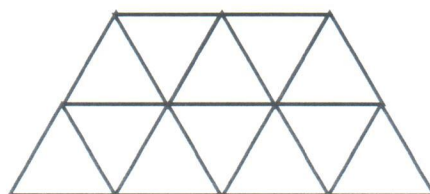
There are a few rules in order to consider a pattern as a regular tessellation.

- Rule 1: The tiles in the tessellation must be regular polygons - and only one kind of polygon for the whole tessellation.
- Rule 2: The tessellation must cover a plane endlessly with no gaps and overlaps.
- Rule 3: Each vertex must have the same number and kind of polygons around it. The sum of the angles at each vertex is 360° , so exactly three regular tessellations exist.



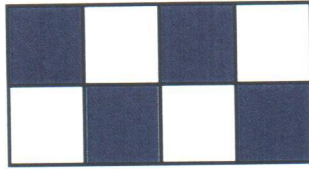
By following these rules, let us identify the regular polygons that can be tessellated.

Tessellation using triangles is possible:

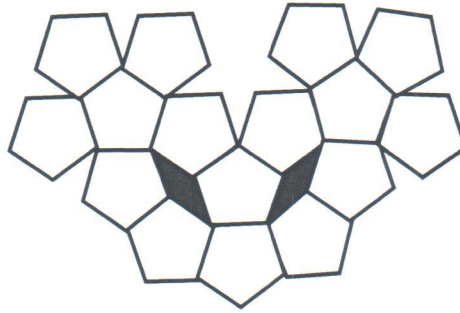




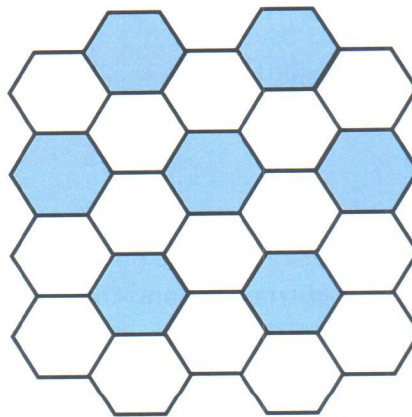
Using squares is possible:



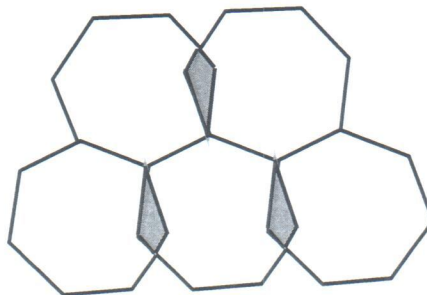
Using pentagons is NOT possible because there are gaps as shown by the shaded areas:



Using hexagons is possible:

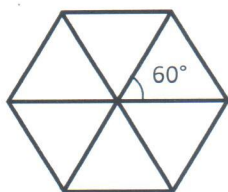


Using heptagons is NOT possible:

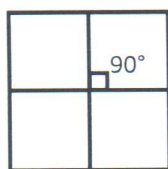


Actually, there are only three regular polygons which can form regular tessellations. These are the **triangle**, **square** and **hexagon**; all of which have interior angles which add up to 360° .

That is, looking at the vertices, we have:



$$60^\circ + 60^\circ + 60^\circ + 60^\circ + 60^\circ + 60^\circ = 360^\circ$$



$$90^\circ + 90^\circ + 90^\circ + 90^\circ = 360^\circ$$



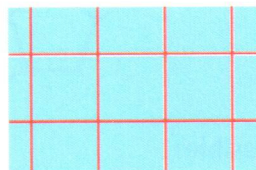
$$120^\circ + 120^\circ + 120^\circ = 360^\circ$$

Here are examples regular tessellations.

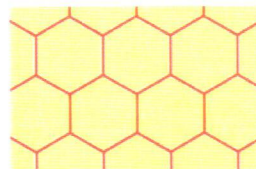
1. a tessellation of triangles



2. a tessellation of squares



3. a tessellation of hexagons

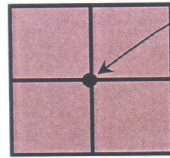


Looking at these three examples you can easily notice that the squares are lined up with each other while the triangles and hexagons are not. Also, if you look at 6 triangles at a time, they form a hexagon, so the tiling of triangles and the tiling of hexagons are similar and they cannot be formed by directly lining shapes up under each other - a slide (or a glide!) is involved.

You can work out the interior measure of the angles for each of these polygons:

Shape	Angle Measure in Degrees
triangle	60
square	90
pentagon	108
hexagon	120
more than six sides	more than 120 degrees

Since the regular polygons in a tessellation must fill the plane at each **vertex**, the interior angle must be an exact divisor of 360 degrees. This works for the triangle, square, and hexagon, and you can show working tessellations for these figures. For all the others, the interior angles are not exact divisors of 360 degrees, and therefore those figures cannot tile the plane.



A Vertex is
where all the
"corners" meet!

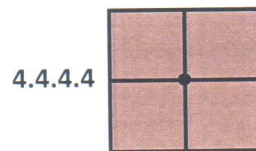
Example 1

A tessellation of squares is named "**4.4.4.4**".

Here is how:

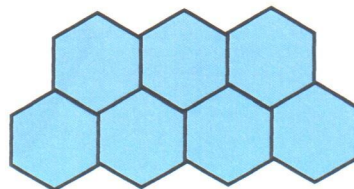
1. Choose a vertex, and then look at one of the polygons that touch that vertex.
How many sides does it have?
Since it's a square, it has four sides, and that's where the first "4" comes from.
2. Now keep going around the vertex in either direction, finding the number of sides of the polygons until you get back to the polygon you started with. How many polygons did you count?
There are four polygons, and each has four sides.
The interior angle of each square is 90.

$$90 + 90 + 90 + 90 = 360 \text{ degrees}$$



Example 2

For a tessellation of regular congruent hexagons, if you choose a vertex and count the sides of the polygons that touch it, you'll see that there are three polygons and each has six sides, so this tessellation is called "**6.6.6**": The interior angle of each hexagon is 120 degrees.

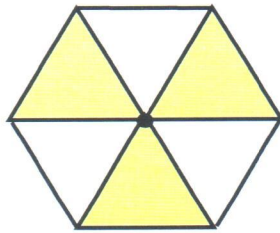


6.6.6

$$120 + 120 + 120 = 360 \text{ degrees}$$

Example 3

A tessellation of triangles has six polygons surrounding a vertex, and each of them has three sides: "**3.3.3.3.3.3**". The interior angle of each equilateral triangle is 60 degrees.



3.3.3.3.3.

$$60 + 60 + 60 + 60 + 60 + 60 = 360 \text{ degrees}$$

For a regular tessellation, the pattern is identical at each vertex.

Now, how do we construct tessellations? Here is an example.

Example 1

Construct a tessellation using regular triangles.

Solution:

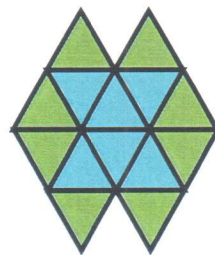
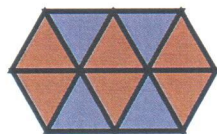
First step is to draw an equilateral triangle like this one.



Next is to draw an invert of the figure adjacent to the first one.



Continue to draw more triangles to form your tessellation pattern. You may also add colours to make your pattern more interesting. These are some examples of the resulting tessellation:



**Learning Activity 12.5.2.1**

20 minutes

A. Use the information in the table below to answer the next two questions.

Regular Polygon (Number of sides)	Size of Interior Angles
Equilateral Triangle (3)	60°
Square (4)	90°
Pentagon (5)	108°
Hexagon (6)	120°
Heptagon (7)	128.6°
Octagon (8)	135°
Nonagon (9)	140°
Decagon (10)	144°

1. Based on the information above, which of the following shapes would not be able to form a regular tessellation?

- a) Equilateral Triangle
- b) Square
- c) Hexagon
- d) Octagon

2. If you were to create a tessellation using 3 of the same shape, which shape from the above list would you use?

- a) Equilateral Triangle
- b) Pentagon
- c) Hexagon
- d) Decagon

B. Circle the letter of the correct answer to each of the following questions.

1. Which of the following do not make a regular tessellation?

- A. Squares
- B. Rectangles
- C. Equilateral triangles
- D. Regular hexagons

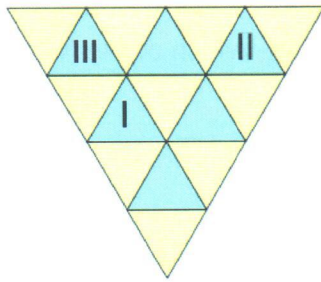
2. Two different pentagons can be used to make:

- A. A regular tessellation
- B. A semi-regular tessellation
- C. A tessellation that is neither regular nor semi-regular
- D. No tessellation at all



3. Squares and rhombuses can be used to make:
- A. No tessellation at all
 - B. A regular tessellation
 - C. A semi-regular tessellation
 - D. A tessellation that is neither regular nor semi-regular
4. Regular hexagons and rhombuses can be used to make:
- A. A tessellation that is neither regular nor semi-regular
 - B. No tessellation at all
 - C. A regular tessellation
 - D. A semi-regular tessellation

C. Use the diagram below to answer the next two questions.



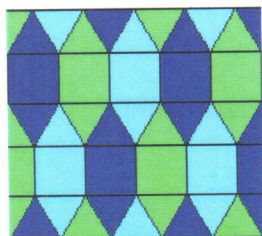
1. For triangle II to be a transformation of triangle I, which of the following transformations could you use? (1 mark)
- a. A translation
 - b. A reflection
 - c. A rotation
 - d. All of the above
2. For triangle III to be a transformation of triangle I, which of the following transformations could you use? (1 mark)
- a. a translation
 - b. a reflection
 - c. a rotation
 - d. Both **b** and **c**

C. Design a tessellation that contains a triangle..

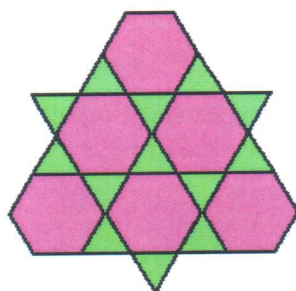
**12.5.2.2: Semi-regular Tessellation**

We can also make a tessellation using a variety of regular polygons.

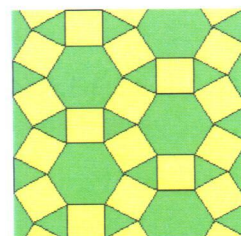
Examples



Squares and triangles



Hexagons and triangles



hexagons, squares, triangles

These tessellation patterns above are all examples of **semi- regular tessellations**.

A semi-regular tessellation is made by using two or more different regular polygons. The pattern at each vertex must be the same!

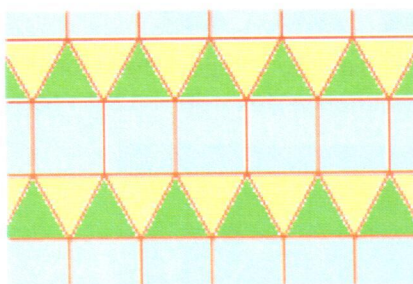
A semi-regular tessellation has two properties which are:

1. It is formed by regular polygons.
2. The arrangement of polygons at every vertex point is identical.

The rules in creating regular tessellations are also applied in creating semi-regular tessellations. Every vertex must have the same configuration or identical.

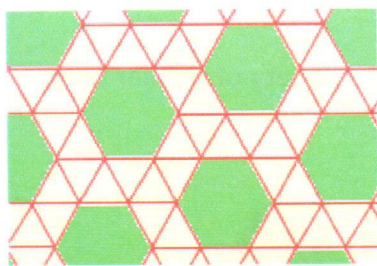
There are only eight semi- regular tessellations. See next page.

Here are the eight Semi-regular Tessellations. Each is shown below with the abbreviated notation signifying how many of which type of regular polygon are located around each common vertex.



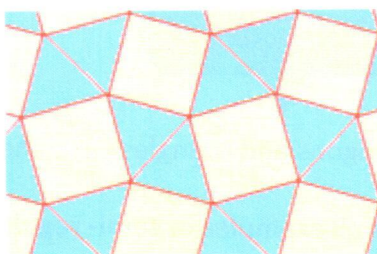
3.3.3.4.4 (3 equilateral triangles and 2 squares)

Angle sum around common vertex = $60^\circ + 60^\circ + 60^\circ + 90^\circ + 90^\circ = 360^\circ$



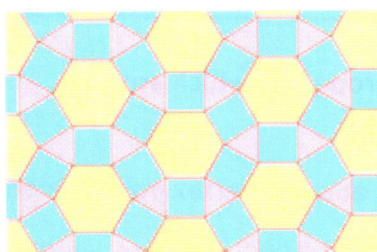
3.3.3.3.6 (4 equilateral triangles and 1 regular hexagon)

Angle sum around common vertex = $60^\circ + 60^\circ + 60^\circ + 60^\circ + 120^\circ = 360^\circ$



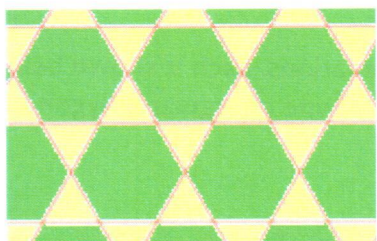
3.3.4.3.4 (3 equilateral triangles and 2 squares)

Angle sum around common vertex = $90^\circ + 60^\circ + 60^\circ + 90^\circ + 60^\circ = 360^\circ$



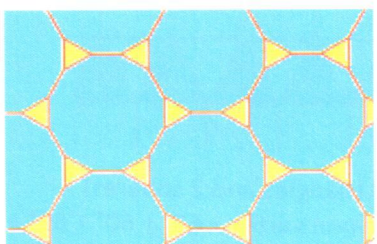
3.4.6.4 (1 equilateral triangle, 2 squares and 1 regular hexagon)

Angle sum around common vertex = $120^\circ + 90^\circ + 60^\circ + 90^\circ = 360^\circ$



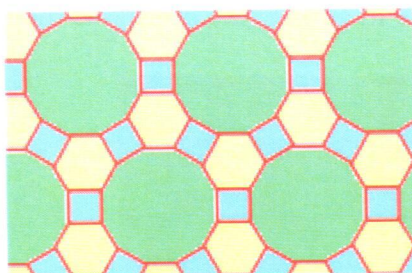
3.6.3.6 (2 equilateral triangles, 2 regular hexagons)

Angle sum around common vertex = $60^\circ + 120^\circ + 60^\circ + 120^\circ = 360^\circ$

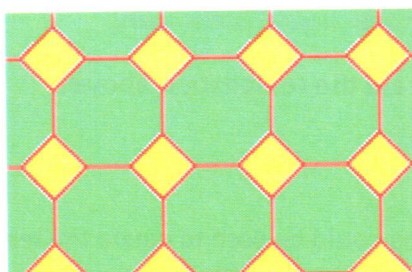


3.12.12 (3 equilateral triangles and 2 dodecagons)

Angle sum around common vertex = $150^\circ + 60^\circ + 150^\circ = 360^\circ$



4.6.12 (1 square, 1 regular hexagon and 1 dodecagon)
Angle sum around common vertex = $150^\circ + 90^\circ + 120^\circ = 360^\circ$



4.8.8 (1 square and 2 regular octagons)
Angle sum around common vertex = $90^\circ + 135^\circ + 135^\circ = 360^\circ$

For each of the semi-regular tessellations above, the sum of the angles around each of the common vertices is 360° and this is the reason why each of these combinations of regular polygons produces a possible tessellation.



Learning Activity 12.5.2.2



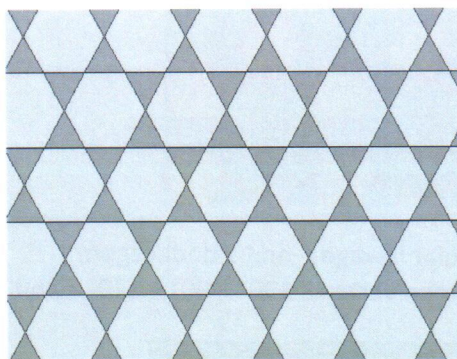
20 minutes

Problem solving

1. Find all the possible semi-regular tessellations that use
 - a. triangles and hexagon
 - b. squares and octagon
 - c. triangles and squares



2. Refer to the tessellation below to answer the questions that follow.



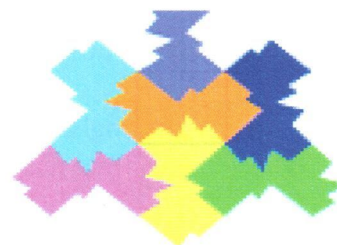
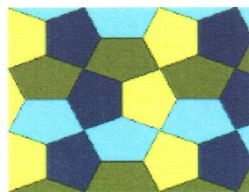
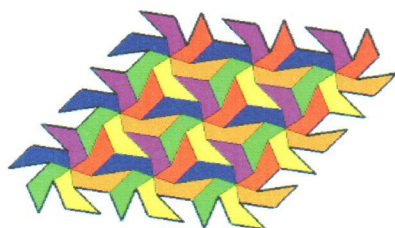
- a. What shapes are used to create the tessellation above?
- b. What type of transformations could be used to create the tessellation above? (1 mark)

12.5.2.3 Non-regular tessellation

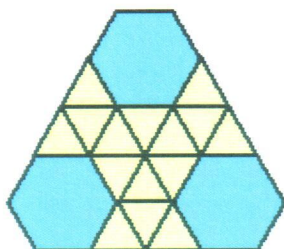
Other combinations of regular polygons do not produce semi-regular tessellations because it is not possible to achieve the angle sum of 360° around each common vertex while maintaining identical tiling around each common vertex.

Non-regular tessellations are those in which there is no restriction on the order of the polygons around vertices.

There is an infinite number of such tessellations. Below are examples of irregular tessellation.

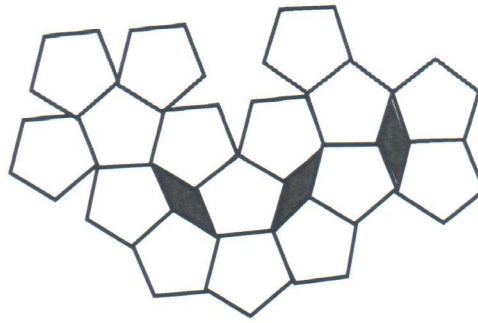


Here's another tessellation made up of hexagons and triangles.



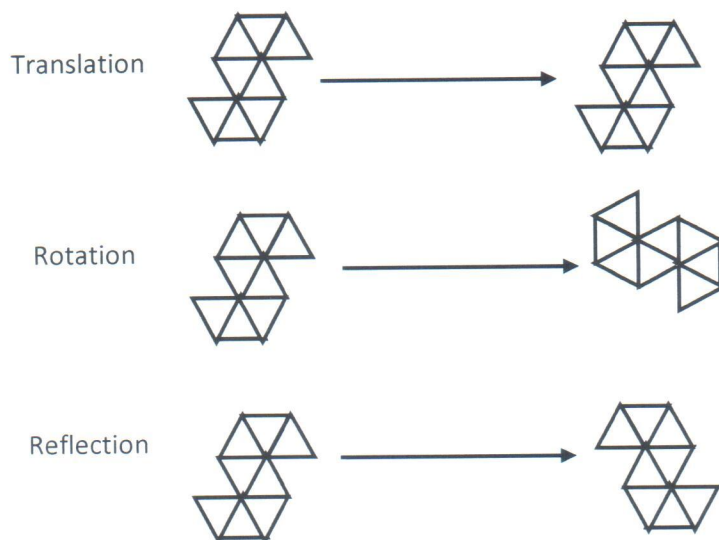
This is not an official semi-regular tessellation because it breaks the rule or pattern at each vertex.

Using pentagons is NOT possible because there are gaps as shown by the shaded areas:



Irregular tessellations of shapes can be created or obtained by performing one or more of three basic operations, translation, rotation and reflection.

See the tessellation using polyiamonds below.



Translation - sliding the polyiamond along the plane. The translation operation can be applied to all polyiamonds.

Rotation - rotating the polyiamond in the plane. The rotation operation can be applied to all polyiamonds which do not possess circular symmetry, for example the hexiamond, which remains unchanged following rotation through 60° or multiples thereof.

Reflection - reflecting the polyiamond in the plane, as if being viewed in a mirror. The reflection operation is limited to polyiamonds which are **enantiomorphic**. An **enantiomorphic polyiamond** is one which cannot be superimposed on its reflection, its mirror image.

Simple tessellations are those in which only the translation operation is used.

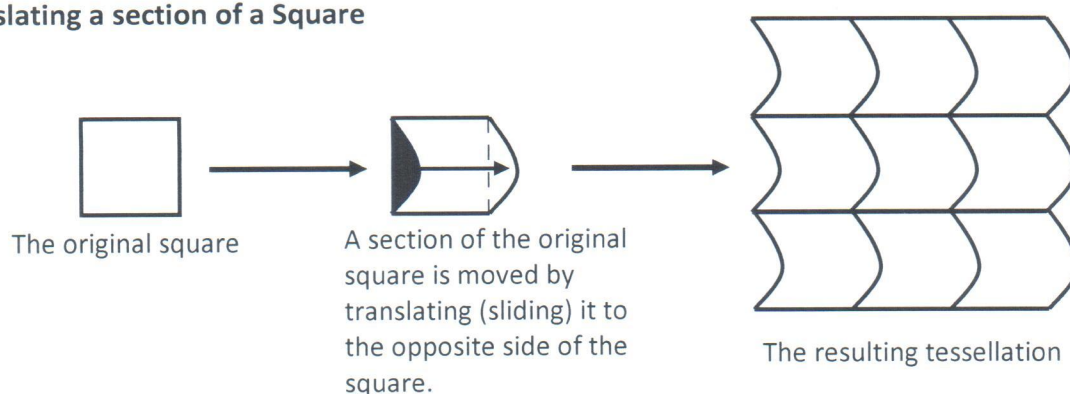
Complex tessellations are those in which one or both of the rotation and reflection operations is used with the translation operation.

It is possible to produce some very irregular shapes which will tessellate by transforming other shapes which are known to tessellate. These irregular shapes include shapes bounded by curved lines.

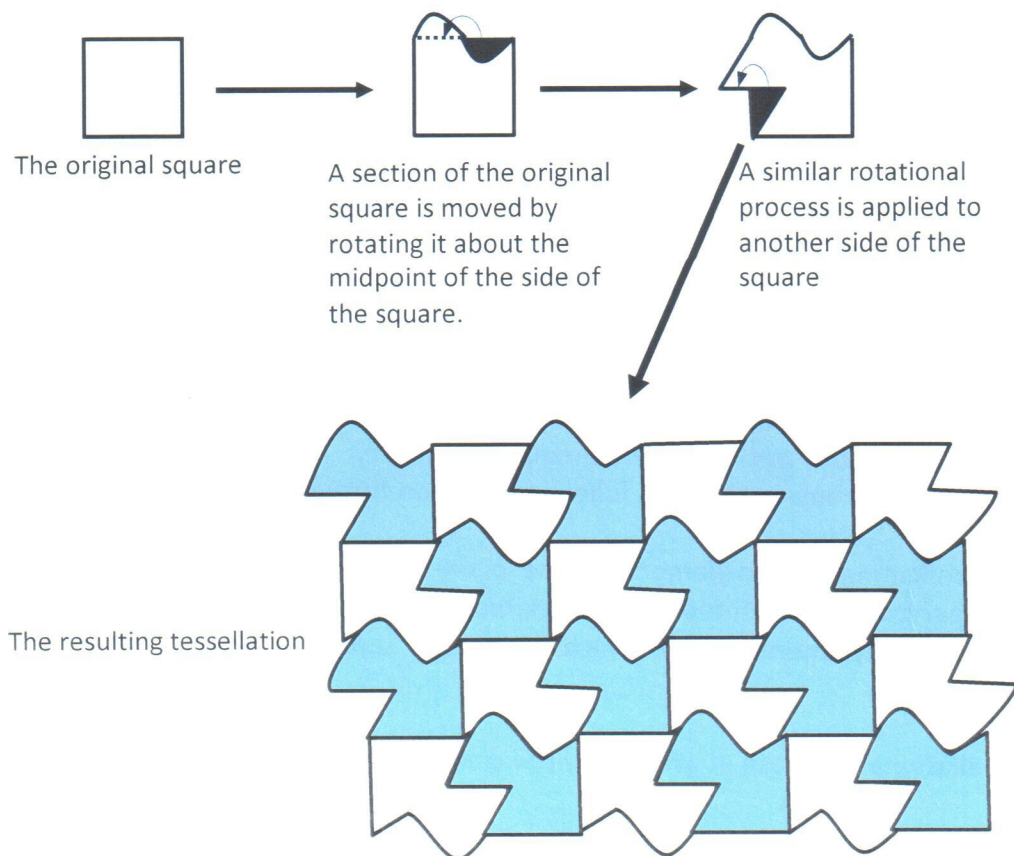
Squares, rectangles, equilateral triangles and hexagons are suitable shapes from which to start. By translating or rotating about any sides sections of the starting shape (or a combination of both) a new, irregular shape can be made which will also tessellate.

Some examples are given below.

Translating a section of a Square



Rotating a section of a Square





B. Write Y if the answer is YES and N if the answer is NO in each of the following questions.

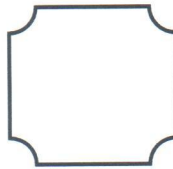
1. Can the shape shown tessellate the plane?



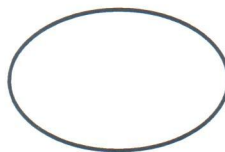
2. Can the shape shown tessellate the plane?



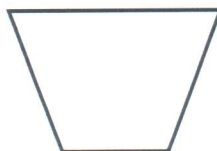
3. Can the shape shown tessellate the plane?



4. Can the shape shown tessellate the plane?



5. Can the shape shown tessellate the plane?



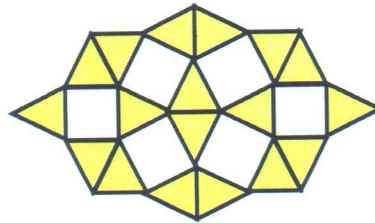
**12.5.2.4. Naming Tessellations**

Now that you are familiar with the different forms of tessellations, it is very important to know how to name them. Naming the regular and semi-regular tessellations is very easy. However, naming irregular tessellations requires creativity.

Some tessellations can be named using number system. You would first pick a vertex in the pattern. Recall that a vertex is a corner of a polygon. It does not matter which vertex you pick. Then identify the polygons around it according to the number of sides each one has.

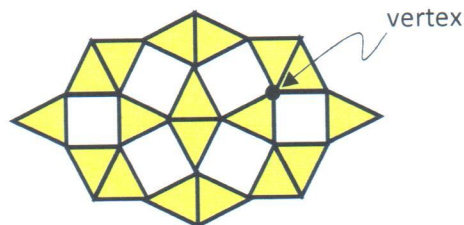
Example

Name this tessellation:

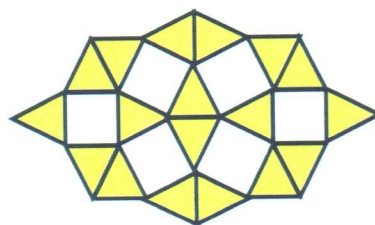


Solution:

First is to identify a vertex. In this case, let us take this one:



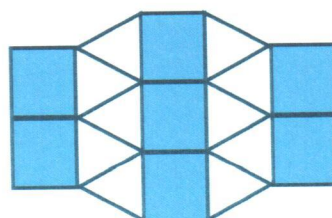
Now, we see that the figures that meet are 3 triangles and 2 squares. To name the tessellation, we begin with the figure with the smallest number of sides. Here, we begin by writing 3. Then we go around the vertex, writing the number of sides of the polygons separated by a comma. So, in this example, the name of the tessellation is 3, 3, 4, 3, 4.



3, 3, 4, 3, 4

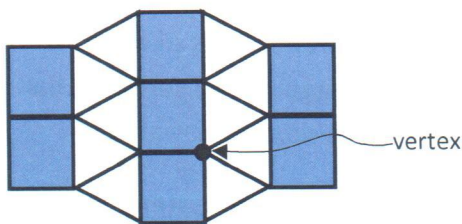
Example 2

Name this tessellation:

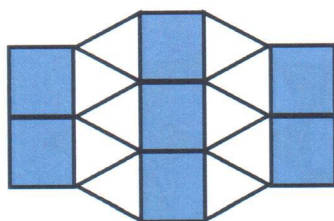




We take a vertex.



To name this tessellation, we begin with the least number of sides. Therefore, this figure is named 3, 3, 3, 4, 4.



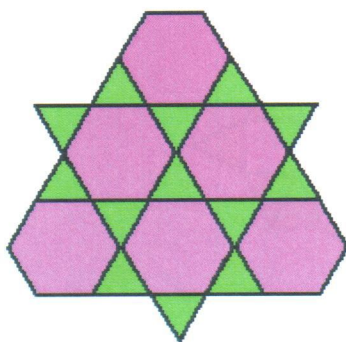
3, 3, 3, 4, 4

Notice that the two examples have the same polygons but are named differently. This shows that the figures are arranged differently in the two tessellations.

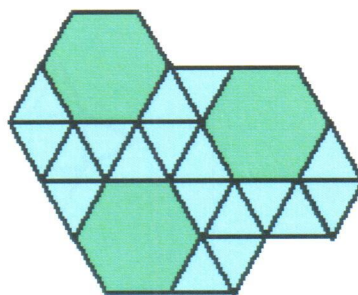
To name a tessellation, simply work your way around one vertex counting the number of sides of the polygons that form that vertex. The trick is to go around the vertex in order so that the smallest numbers possible appear first.

Here are other examples below.

These tessellations are both made up of hexagons and triangles, but their vertex configuration is different. That's why we've named them!



3.6.3.6



3.3.3.3.6

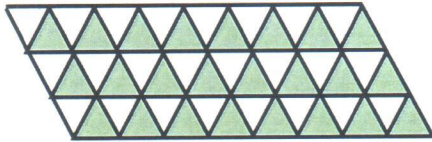
That's why we wouldn't call our 3, 3, 3, 3, 6 tessellation a 3, 3, 6, 3, 3!

Now do the learning activity.

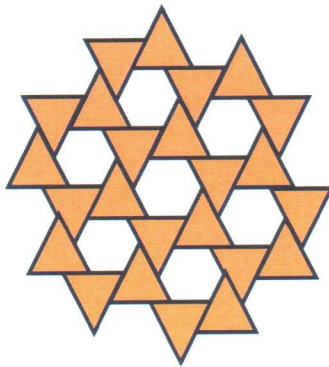
**Learning Activity 12.5.2.4****20 minutes**

Name the following tessellations using number system.

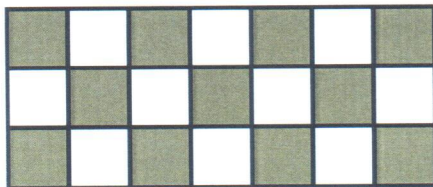
1.



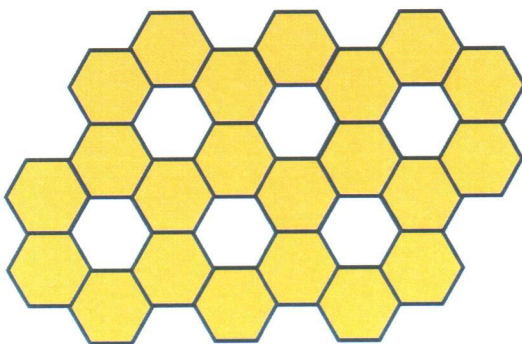
2.



3.



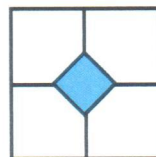
4.



**SUMMATIVE TASK 12.5.2**

20 minutes

- A. Choose the correct answer from the alternatives A, B, C and D. Circle the letter of the your answer.
- A tessellation can be most accurately described as _____.
 - a collection
 - tiling
 - a collage
 - patchwork
 - A tessellation is created using one repeating square. Which type of tessellation is this?
 - Irregular
 - symmetric
 - regular
 - semi-regular
 - Which of the following is NOT true of tessellations?
 - Any shape can be used.
 - The shapes can overlap.
 - More than two shapes can be used.
 - There can be no spaces between the shapes.
 - We can tile a plane with _____.
 - an equilateral triangle
 - an isosceles Triangle
 - a scalene triangle
 - all of these
 - How many polygons are used to create the tessellation below?
 - 2
 - 3
 - 4
 - 5
 - What polygons were used to create the tessellation shown?

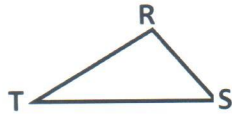


- 4 regular pentagons and a square
- 1 regular pentagon and a square
- 2 irregular pentagons and a square
- 4 irregular pentagons and a square

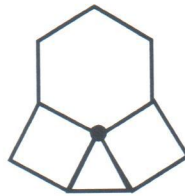


B. Answer the following.

1. Find all the possible semi-regular tessellations that use squares and octagons.
2. Create a tessellation with triangle **RST**.



3. Use each arrangement of regular polygons to create a semi-regular tessellation.



4. Use each shape to create a tessellation.





12.5.3 APPLICATION OF PATTERNS AND SHAPES IN TRADITIONAL ARTS

Geometry basically deals with patterns, shapes and lines. It was used traditionally by the Egyptians and other ancient countries in helping to design buildings and to create design them. In Papua New Guinea our people do use geometry in their traditional art work without realizing that they are using it. However, their understanding of this art work is simply making patterns.

Culturally, in Papua New Guinea societies, patterns come in different styles and forms but do not vary very much. It is of great importance to different ethnic groups because it represents the people's beliefs, feelings and creativity in the form of art. It gives them pride and respect for their tradition and culture.

Traditionally, in Papua New Guinea societies, patterns are formed by repeated shapes clustered together or arranged systematically. This is simply using congruent shapes to make or create patterns.

Papua New Guinean tribes have been using tessellations in their art since ancient times. These intricate patterns have been created with the use of natural pigments on different canvass. Such designs can be found on ceremonial shields, bilums, dresses, masks and among others.

12.5.3.1 Patterns and Shapes in Traditional Arts

Having covered polyhedral and tessellations in the first and second topics, this topic looks at how mathematical shapes are applied in the traditional patterns and measurement in Papua New Guinea and use mathematical language to describe them.

For example, look at the following decorative patterns of work of art and describe them in terms of polyiominoes, polyiamonds or other tessellations.

Can you identify the polygons used in creating these patterns?

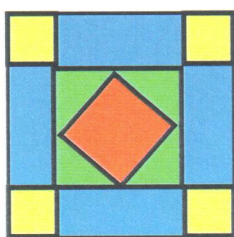


Figure 1



Figure 2

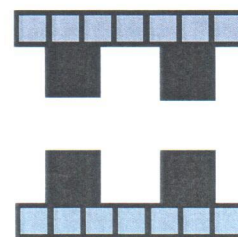


Figure 3

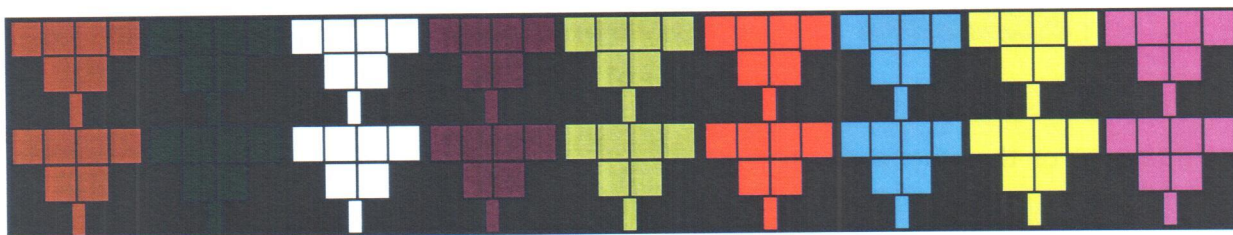
Analysing Patterns

Example 1

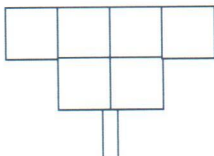
Here is an image of a hand-crafted native bag (bilum) made using tessellations. Now, we will analyse the figure used.

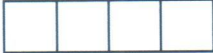


Let us have a closer look of the pattern. The main pattern in the bilum is reproduced below.

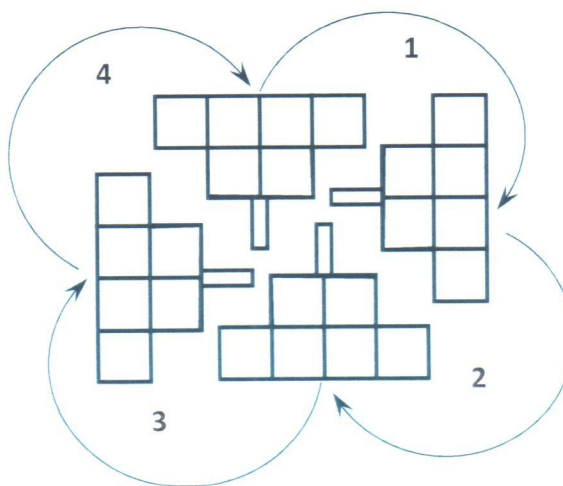
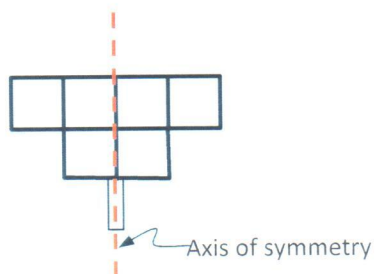


Notice that the pattern of the bilum is a combination of a rectangle and polyominoes.

The dominant pattern  is a combination of

a rectangle  , a domino  and tetromino 

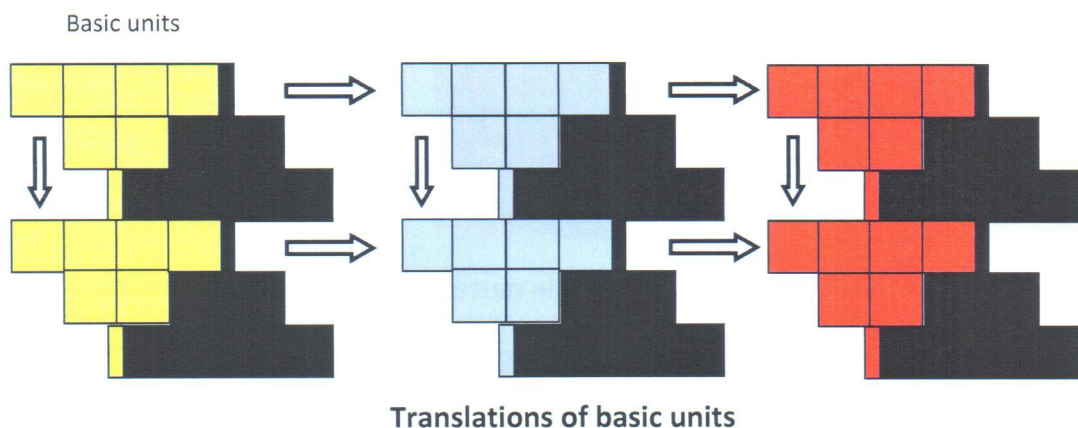
The dominant pattern has one axis of symmetry and has rotational symmetry of order four.



Rotational symmetry of order 4

The 'fill in' area (dark area) is a rotation of the coloured pattern of polyominoes.

The basic unit of pattern (coloured pattern) is given. There is only one axis of symmetry for the basic unit of pattern and to tile the plane translations of the basic unit of pattern are required.

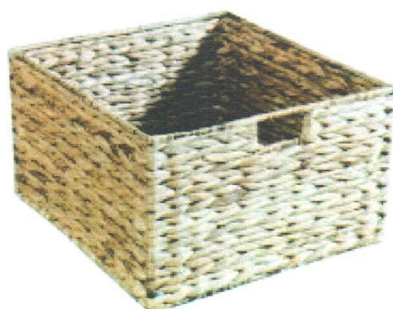


Further work on the other patterns on the bilum(bag) and handle could be done. The volume or total surface area could also be investigated.

Analysing Shapes

Example 2

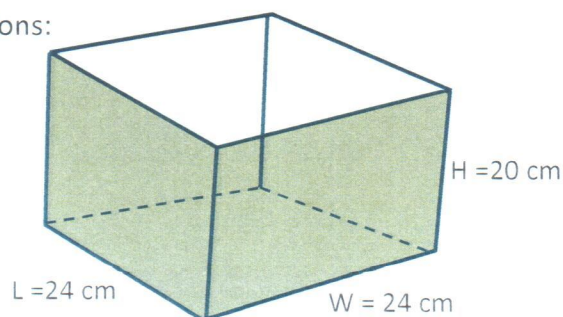
Consider the basket below.



The basic shape of this basket is that of a rectangular prism or a cuboid. A rectangular prism is a prism whose faces are all rectangles.

Measuring the basket gives the following dimensions:

Length = 24 cm
Width = 24 cm
Height = 20 cm





The capacity (volume of the basket can be found by using the formula: $\text{Volume} = L \times W \times H$

Hence, the Volume of the basket = $(24 \text{ cm})(22 \text{ m})(20 \text{ cm})$
 $= 10\,560 \text{ cm}^3$

Therefore, the capacity (volume of the basket is $10\,560 \text{ cm}^3$ or 10.6 litres. (Since, there are 1000 cm^3 in one litre.)

Since the basket is open, the Total surface area is obtained by the formula:

$\text{TSA} = \text{area of the base} + \text{areas of four side faces}$

$$\begin{aligned} &= (24 \text{ cm})(24 \text{ cm}) + 4[(24 \text{ cm})(20 \text{ cm})] \\ &= 576 \text{ cm}^2 + 4(480 \text{ cm}^2) \\ &= 576 \text{ cm}^2 + 1920 \text{ cm}^2 \\ &= 2496 \text{ cm}^2 \end{aligned}$$

Example 3

Consider the basket on the right.



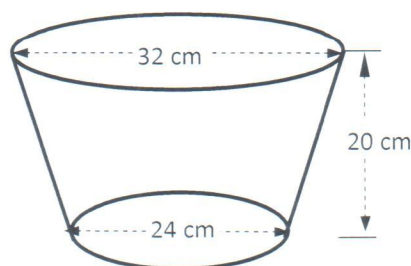
The basic shape of this basket is that of a truncated right cone. A truncated right cone has a centre of the upper circular edge vertically above the centre of the circular base.

Measuring the basket gives the following dimensions:

Diameter of the circular edge is 32 cm.

Diameter of the base is 24 cm

Height of the basket is 20 cm.



To find the volume or capacity of the basket we use the formula:

$$V = \frac{1}{3} (B + B_1 + \sqrt{BB_1}) \times H$$

Where: B = area of the lower base

B_1 = area of the upper base

H = is the vertical height of the basket (truncated cone)

Since the bases are circular, to find their areas we use the formula $A = \pi r^2$.



To find the area of the lower base (B), we have: $A = \pi r^2$

$$\begin{aligned} &= 3.14(12 \text{ cm})^2 \\ &= 3.14 \times 144 \text{ cm}^2 \\ &= 452.16 \text{ cm}^2 \end{aligned}$$

To find the area of the upper base (B_1), we have: $A = \pi r^2$

$$\begin{aligned} &= 3.14(16 \text{ cm})^2 \\ &= 3.14 \times 256 \text{ cm}^2 \\ &= 803.84 \text{ cm}^2 \end{aligned}$$

Substituting these in the formula for the volume: $V = \frac{1}{3}(B + B_1 + \sqrt{BB_1}) \times H$

We have: $V = \frac{1}{3}[452.16 \text{ cm}^2 + 803.84 \text{ cm}^2 + \sqrt{(452.16 \times 803.84)}](20 \text{ cm})$

$$\begin{aligned} &= \frac{1}{3}(1256 + \sqrt{363464.2944})(20) \text{ cm}^3 \\ &= \frac{1}{3}(1256 + 602.88)(20) \text{ cm}^3 \\ &= \frac{1}{3}(1858.88)(20) \text{ cm}^3 \\ &= \frac{1}{3}(37\,177.6) \text{ cm}^3 \\ &= 12\,392.53 \text{ cm}^3 \end{aligned}$$

Hence the capacity of the basket is 12.392 or approximately 12.4 litres.

Now do the learning activity on the next page.

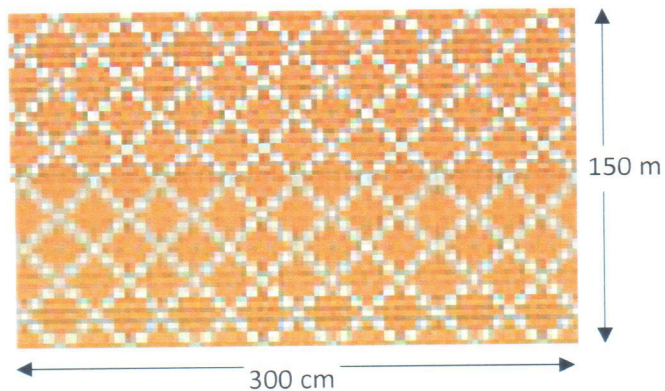
**Learning Activity 12.5.3.1**

20 minutes

1. Apply your knowledge of shapes and patterns to analyse the shape of this basket and used mathematical terms to describe it. The shape is that of a cylinder where the diameter of the circular base is 30 cm and the height is 36 cm.



2. Apply your knowledge of shapes and patterns to analyse this mat.





UNIT SUMMARY

This summary outlines the important key ideas and concepts to be remembered.

12.5.1: Polyhedra

- **Polyhedra** are solids that have polygons as their faces.
- **Prisms** are solids that have a constant cross-section that is a polygon with two identical bases and rectangular sides. Their names are derived from its cross-section.
- The **cross section** of a prism is the shape made by cutting straight across an object.
- **Pyramid** is a solid whose base is a polygon and whose sides are triangles attached to one point called the vertex or apex.
- A **right pyramid** is a pyramid whose vertex is directly above the centre of the base.
- A **non-right pyramid** is a pyramid whose vertex is not vertically above the centre of the base.
- Pyramids are named after the type of bases they have.
- **Platonic solids** are polyhedra whose faces are regular polygons with congruent sizes wherein the same number of faces are connected at the vertex.
- There are only five platonic solids:
 1. **Tetrahedron** is a polyhedron with four (4) faces that are equilateral triangles. It has six (6) edges and four (4) vertices and at each vertex three triangles meet.
 2. **Cube** is a polyhedron with six (6) faces that are squares. It has 12 edges and 8 vertices and at each vertex three squares meet.
 3. **Octahedron** is a polyhedron with eight (8) faces that are equilateral triangles. It has 12 edges and 6 vertices and at each vertex four triangles meet.
 4. **Dodecahedron** is a polyhedron with twelve (12) faces that are regular pentagons. It has 30 edges and 20 vertices and at each vertex three pentagons meet.
 5. **Icosahedron** is a polyhedron with twenty (20) faces that are equilateral triangles. It has 30 edges and 12 vertices and at each vertex five triangles meet.
- The **Polyhedron Formula** or commonly known **Euler's Formula** states that:
"The number of faces (F) plus the number of vertices (V) minus the number of edges (E) equals 2". In symbol it is written as: $F + V - E = 2$
- **Symmetry** is when one shape becomes exactly like another if you flip, slide or turn it.
- There are two kinds of symmetry:
 1. **Line or Reflectional Symmetry** exists when it is possible to draw a line through the figure so that one side is a mirror image of the other. The line is called the **axis** or **line of symmetry**.
 2. **Point or Rotational Symmetry** exist when a shape remains the same when rotated through any angle less than 360 degrees. The number of distinct ways which a shape or figure can map onto itself by rotation is called the **order of rotational symmetry**.
- **Polyominoes** are shapes that are made up of squares joined edge to edge.
- **Polyiamonds** are shapes that are made up of equilateral triangles joined edge to edge.



- Two polyominoes or two polyiamonds are the same if they can be translated (slide) sideways, or up and down, reflected (as in a mirror) or rotated (turned around) so that they look the same.

12.5.2: Tessellation

- Tessellation is a pattern of polygons, each of which fit together around a common point called vertex. It simply means “tiling”.
- **Regular tessellation** is a tessellation made up of congruent regular polygons. The tessellation covers a plane without gaps and overlaps.
- There are only three (3) regular polygons that can be tessellated. These are the **triangle, square and hexagon**.
- **Semi-regular tessellation** is made by using two or more different regular polygons. The pattern at each vertex must be the same. There are only eight semi-regular tessellations.
- **Non-regular or irregular tessellations** are those in which there is no restriction on the order of the polygons around vertices. There is an infinite number of non-regular tessellations.
- Tessellations can be created by using one or more of the three basic operations:
 1. **Translation** – sliding the shape along the plane
 2. **Rotation** – rotating the shape in the plane.
 3. **Reflection** – reflecting the shape in the plane as if being viewed in a mirror.

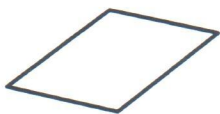
12.5.3: Application of Geometry in PNG Arts

- Geometry has a lot of practical uses, from the most basic to the most advanced phenomena in life. Even the very basic concept of area can be a huge factor in how you do your daily business.

REVISE SECTIONS 12.5.1 TO 12.5.3 THEN DO MODULE ASSESSMENT 5.
--

**ANSWERS TO LEARNING ACTIVITIES 12.5.1 to 12.5.3****Learning activity 12.5.1.1**

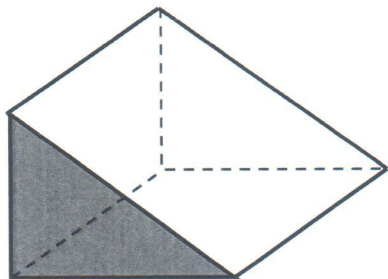
1. a.



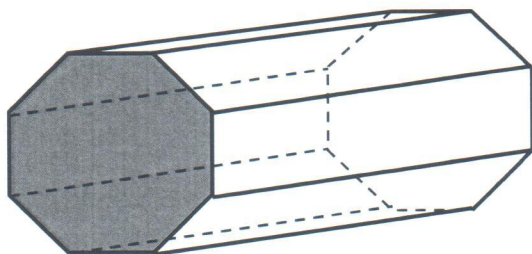
b.



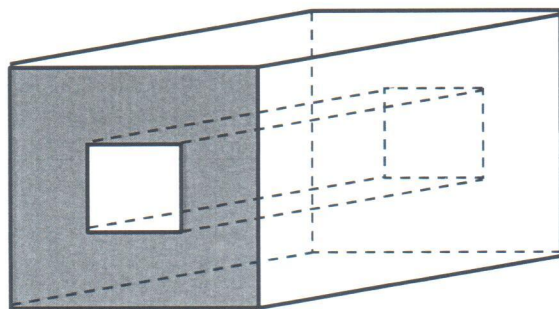
2. a.



b.

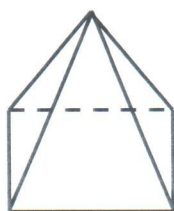


c.

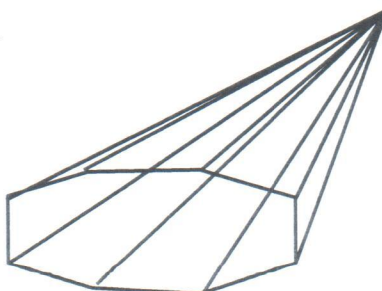
**Learning Activity 12.5.1.2**

1. a. P b. NP c. NP

2. a.



b.



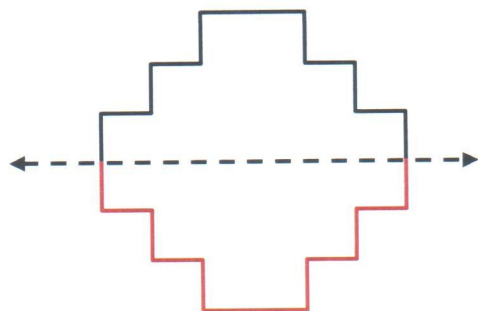
**Learning activity 12.5.1.3**

A. 1. C 2. D 3. A 4. D 5. B

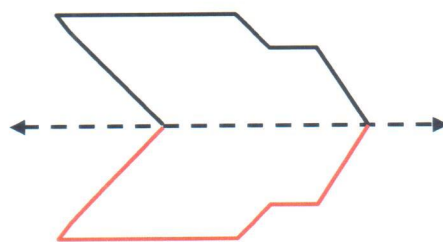
- B. 1. Cube
2. octahedron
3. dodecahedron
4. tetrahedron
5. icosahedron

Learning activity 12.5.1.4

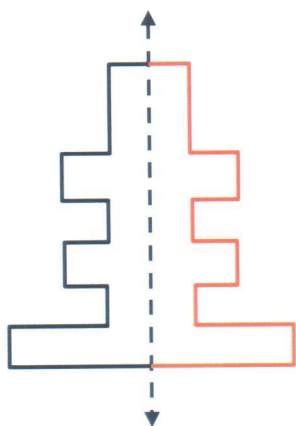
1. a.



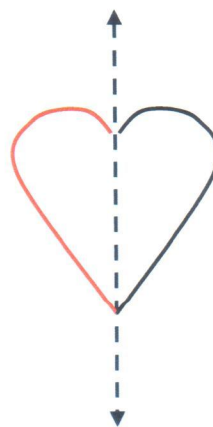
b.



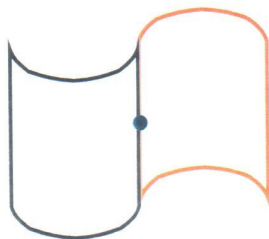
c.



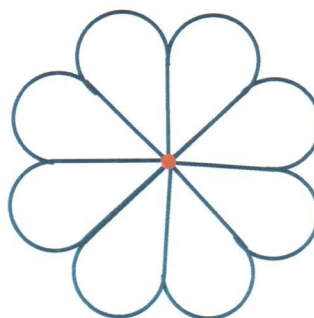
d.



2. a.



b.



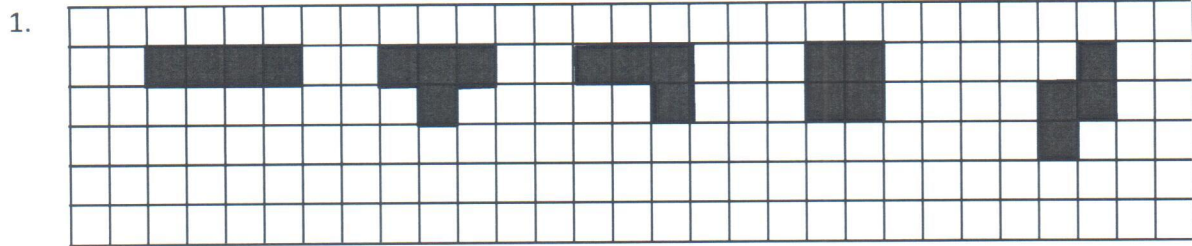
3. a. number of lines of symmetry = 3
b. order of rotational symmetry = 3



4. a) 1 b) 2 c) 4 d) 6
e) 8 f) 2 g) 4 h) 5
i) 2 j) 4 k) 3 l) 8
m) 4 n) 1 o) 2 p) 2

5. a, b, and c

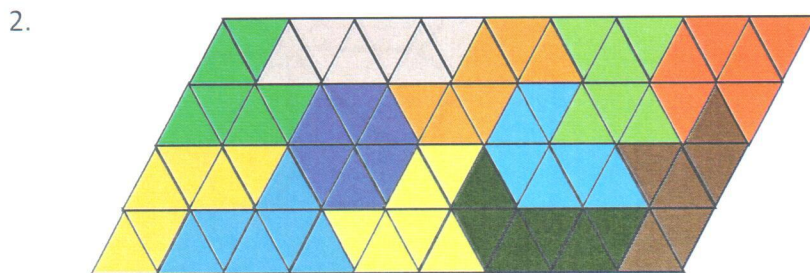
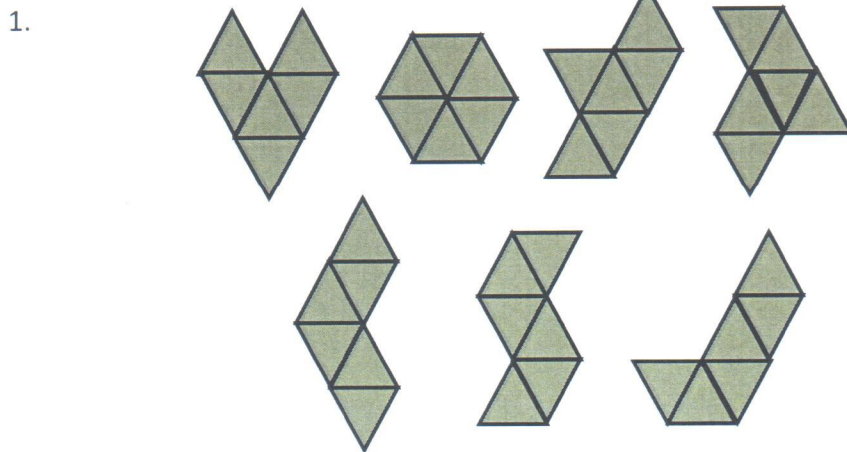
Learning activity 12.5.1.5



2. A and O, B and P, E and M, C and J, D and N, F and K, G and L, H and P

3. a. $P = 14 \text{ cm}$
 $A = 6 \text{ cm}^2$ b. $P = 10 \text{ cm}$
 $A = 5 \text{ cm}^2$ c. $P = 8 \text{ cm}$
 $A = 3 \text{ cm}^2$ d. $P = 12 \text{ cm}$
 $A = 5 \text{ cm}^2$

Learning activity 12.5.1.6



**SUMMATIVE TASK 12.5.1**

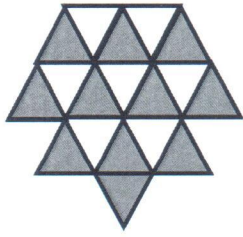
- A.
1. 8, 12, 6
 2. vertex
 3. Prism
 4. Tetrahedron
 5. Regular
 6. Pyramid
 7. Prism
 8. Euler's Formula
 9. Polyominoes
 10. Polyiamonds
- B.
- | | | | | |
|----------------|--------|----------|--------|-------|
| 1. (i) 6 | (ii) 6 | (iii) 5 | (iv) 5 | (v) 4 |
| 2. (i) 6 | (ii) 8 | (iii) 12 | (iv) 9 | |
| 3. (i) 8 | (ii) 5 | (iii) 4 | (iv) 6 | |
| 4. (i) square | | | | |
| (ii) triangle | | | | |
| (iii) triangle | | | | |
| (iv) triangle | | | | |
| (v) pentagon | | | | |
- C.
- | | | | | | |
|------|------|------|------|------|------|
| 1. 1 | 2. 6 | 3. 5 | 4. 3 | 5. 3 | 6. 4 |
|------|------|------|------|------|------|
- D.
- | | | | | |
|------|------|------|------|------|
| 1. D | 2. A | 3. D | 4. C | 5. B |
|------|------|------|------|------|

Learning activity 12.5.2.1

- A.
1. Octagon
 2. hexagon
- B.
1. B
 2. C
 3. D
 4. A
- C.
1. A
 2. D

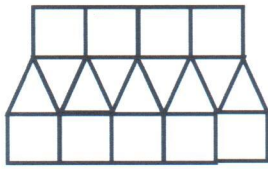


D.

**Learning activity 12.5.2.2**

1.
 - a. There are exactly two semi-regular tessellations that use triangles and hexagons.
 - b. There is exactly one semi-regular tessellation that use squares and octagons.
 - c. There are exactly two semi-regular tessellations that use triangles and squares.
2.
 - a. triangles and hexagons
 - b. reflection and rotation

3.

**Learning activity 12.5.2.3**

- A.
 1. D
 2. tessellation
 3. semi-regular tessellation
 4. tiling
 5. A
- B.
 1. Y
 2. N
 3. N
 4. N
 5. Y

Learning activity 12.5.2.4

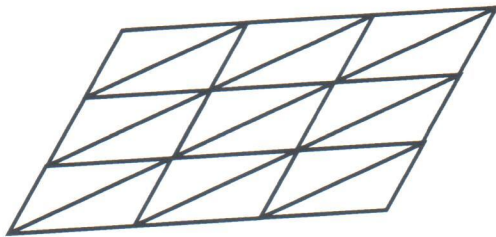
1. 3, 3, 3, 3, 3, 3 tessellation
2. 3, 3, 6 tessellation
3. 4.4.4.4 tessellation
4. 6.6.6 tessellation

**Summative task 12.5.2**

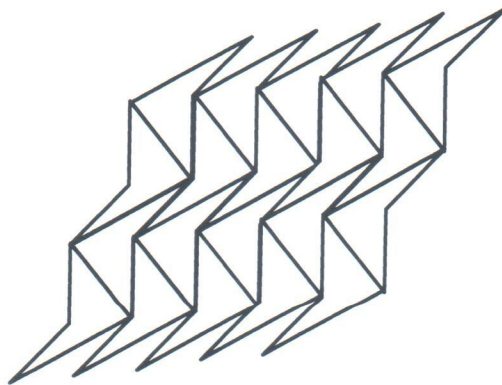
- A. 1. B
2. C
3. B
4. D
5. D
6. D

- B. 1. There is only one possibility: a square and 2 octagons

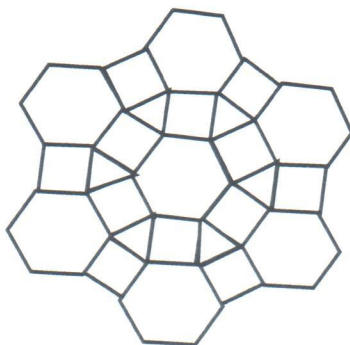
2.



3.

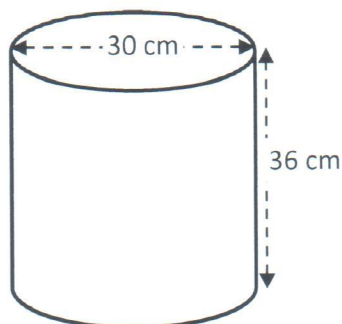


4.



**Learning activity 12.5.3.1**

1. The shape of the basket is that of a right cylinder.



The formula for the volume of the cylinder is:

$$V = \pi r^2 h$$

Since the diameter of the base is 30 cm, then the radius is 15 cm.

$$V = 3.14(15 \text{ cm})^2(36 \text{ cm})$$

$$V = 3.14(225)(36) \text{ cm}^3$$

$$V = 3.14(8100) \text{ cm}^3$$

$$V = 25\,434 \text{ cm}^3$$

Therefore, the capacity or volume of the basket is 25.434 cm^3 or approximately 25.4 litres.

2. The shape of the basket is that of a rectangle.

The basic pattern of the mat is a combination of monominoes.



The dominant patterns are



and



which may vary

depending on other's perspective. Hence, answers may vary.

The area of the mat is found by using the formula $A = l \times w$

$$A = 450 \text{ cm} \times 250 \text{ cm}$$

$$= 112\,500 \text{ cm}^2$$

END OF UNIT MODULE 12.5



REFERENCES

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- Maths for Qld 1 and 2, by: Paul Burns and Chris Lynagh
- School Mathematics of East Africa, Book 2; East African Publishing House and Cambridge University Press
- Oxford; New Edition: Extended Mathematics for IGCSE; David Rayner
- <https://www.mathsisfun.com/definitions/tessellation.html>
- www.coolmath.com/lesson-tessellations-1

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Grade Levels	Subjects
Grades 7 and 8	1. English
	2. Mathematics
	3. Science
	4. Social Science
	5. Making a Living
	6. Personal Development
	7. English
Grades 9 and 10	1. English
	2. Formal Mathematics
	3. Practical Mathematics
	4. Science
	5. Social Science
	6. Commerce
	7. Design and Technology- Computing
	8. Personal Development
Grades 11 and 12	1. English
	• Applied English
	• Language and Literature
	2. Mathematics
	• Mathematics A
	• Mathematics B
	3. Science
	4. Social Science
	• History
	• Geography
	• Economics
	5. Business Studies
	6. Personal Development
	7. ICT

REMEMBER:

- For Grades 7 and 8, you are required to do all six (6) courses.
- For Grades 9 and 10, you must study English, Mathematics, Science, Personal Development, Social Science and Commerce. Design and Technology-Computing is optional.
- For Grades 11 and 12, you are required to complete seven (7) out of thirteen (13) courses to be certified. Your Provincial Coordinator or Supervisor will give you more information regarding each subject.

Certificate in Matriculation

CORE COURSES

Basic English
English 1
English 2
Basic Maths
Maths 1

Maths 2

History of Science & Technology

OPTIONAL COURSES

Science Streams: Biology

Chemistry, Physics and Social Science Streams:

Geography, Introduction to Economics and Asia and the Modern World

REMEMBER:

You must successfully complete 8 courses; 5 compulsory and 3 optional